Airborne Radar Retrieved 3D Wind Fields for Turbulence Detection

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1. Introduction

Atmospheric Turbulence is a frequent cause of injury on board in airliners. Strong turbulences occur inside convective cells. In most cases, a reflectivity map provided by an airborne Weather X-band Radar (WXR) is enough to avoid these areas. However, turbulence may be also encountered in areas outside the largest reflective core: indeed, vertical air motions induce gravity waves that propagate far away from the convective core. This kind of turbulence is not detectable by means of reflectivity, and the detection of such turbulence can only rely on the measurement and the analysis of the motion of the atmosphere. Up to now, the turbulence warnings have been obtained using Doppler Radars and are based on the measurement of statistical properties of the radial component of the wind, measured nearly horizontally, ahead the aircraft. Some hypotheses, such as the isotropy of the turbulence, are made to estimate the other components: indeed, “en-route” aircrafts are mainly disturbed by vertical and lateral turbulences. However, these hypotheses are not always fulfilled. The proposed method does not use these “statistical” radial properties of the wind field, but performs a full 3D reconstruction, without a priori assumption on the wind field.

Basically, the proposed method performs a best fit between a set of radial measurements and projections of an analytical 3D model of the air motion on these Lines of Sight. The unknowns to be found are the parameters of the model. This method uses a variational formalism (i.e. a least-squares error minimization between Doppler measurements and the analytical velocity model). Without any constraint, this inverse problem is ill-posed because only a set of quasi-co-linear radial measurements is available to reconstruct the three velocity components of the wind field. To close the problem, two physical constraints are taken into account: The air mass conservation (continuity equation) and the water or ice mass continuity equation. These constraints are globally embedded in the global minimization. In the future, this method could provide detection of Clear Air Turbulence by inference; even help mitigate the effects of turbulence by coupling the radar to the flight control computer for 3D wind field retrieval. This method is referred hereafter as DTCOR (Detection of Convector Turbulence by Radar).

Section 2 describes the general principles of the proposed method for 3D wind field retrieval, section 3 gives more details on the DTCOR and section 4 discusses the experimental results. Finally, conclusions are given in section 5.

2. The Proposed method for 3D wind field retrieval

2.1 General Principles

A Doppler radar can measure only radial velocity components along Lines Of Sight (LOS). Basically, the DTCOR method performs a best fit between a set of radial measurements: \( V_r \) (\( i \in [1,N] \)) and projections of an analytical 3D model of the air motion on these LOS: \( V_{ra} \). The unknowns to be found are the parameters of the model. This method uses a variational formalism (i.e. a least-squares error minimization between Doppler measurements and the analytical velocity model):

\[
S = \frac{1}{N} \sum_i (V_{ra_i} - V_r) \quad \text{minimum}
\]

(1)

Without any other constraint, this inverse problem is ill-posed because only a set of quasi-colinear radial measurements is available to reconstruct the three velocity components of the wind field: \( \{U_i, V_i, W_i\} \). At this point, the fact that the echoes come from a fluid obeying physical rules has not been taken into account. Two of the rules are particularly significant:

1) Air mass conservation (continuity equation):
2) Water or ice mass continuity equation

These constraints are globally embedded in a global minimization of least-squares sense with the weights \( \lambda_0 \), \( \lambda_1 \) and \( \lambda_2 \). The physical constraints are not fulfilled at each point of measurement, but are verified in a statistical sense on the set of \( N \) measurements:

\[
\lambda_0 S + \lambda_1 S_1 + \lambda_2 S_2 \quad \text{minimum}
\]

(2)
The expressions of $S_1$ and $S_2$, which are respectively related to the two constraints, will be provided hereafter. An estimate of the weighting factors is made by attributing a confidence inversely proportional to the quadratic error on $S_1$ and $S_2$ (Scialom and Lemaître, 1990) to each constraint. With the addition of these constraints, the problem may be well conditioned if some rules on spatial sampling are fulfilled: indeed the problem remains ill-conditioned, despite of the constraints, if all LOS are collinear.

2.2 Spatial Sampling Strategy

The Radar measurements are analyzed in a 3D domain called “$D$” inside a scanning region “$D$”, located ahead of the A/C. A 2D scanning strategy is used, both in azimuth and elevation. The method may be applied simultaneously on several adjacent “$D$” domains (within D) to get a sufficient wide warning domain (cf. Fig. 1). In this case, a multi-scale approach is used to avoid discontinuities between adjacent reconstruction domains. The domain “$D$” is preferably centered on the foreseen trajectory if detectable tracers are present (as represented in Fig. 1(a)).

![Fig. 1 (a) Retrieval Domain “$D$” - scanned Domain “$D$”, (b) A/C motion to increase angular diversity (Own ship reference)](image)

The domain “$D$” must be large enough to provide a sufficient number of independent Radar measurements (corresponding to separate “Range – Angle” resolution cells) with a sufficient LOS diversity to avoid an ill-conditioned problem. However, this area must also be limited for several reasons. First, the wind field model should be valid within “$D$”. Second, a large LOS diversity in elevation is not possible with airborne Radar: Negative elevations are limited by the ground clutter which increases as tilt decreases, and positive elevations are limited as well by the lack of detectable tracers at high altitudes.

The two opposite conditions on the “$D$” size cannot be fulfilled with a single scan. Therefore, the A/C motion is used to increase both the number of measurement points and the angular diversity of the observations. During the A/C motion, the reconstruction domain “$D$”, which is fixed with respect to ground reference, moves with respect to the domain “$D$”, associated with the A/C. Fig. 1(b) illustrates this sampling strategy with respect to the own aircraft reference. Fig. 2 highlights the same idea, but in reference to the ground.

![Fig. 2 Use of the A/C motion to increase angular diversity and the number of sampling points (Ground reference)](image)

This sampling process is repeated from sweep to sweep. For instance, at four times T1 to T4, the radial velocity of air at a point M is measured at four different LOS. The reconstruction problem becomes progressively well-conditioned while the domain “$D$” gets closer to the A/C and the number of measurements at various LOS increases. The wind field (in an Eulerian sense) may be assumed as static during a time period of a few minutes. In others words, the velocity components depend only on space coordinates and not on time. The successive reflectivity maps can be correlated in order to compensate for slight effects from advection.

3. The DTCOR Method

This method consists in retrieving the model parameters. Three wind field models have been experienced:

1) The linear model where the wind divergence and the vertical wind are constant in the horizontal plane.
2) The quadratic model where the wind divergence and the vertical wind are linear in the horizontal plane.
3) The sinusoidal model, where each velocity component $V_i$ can be expressed as products of three sinusoidal functions, each of them depending of a spatial coordinate $x$ or $y$ or $z$.

The method consists of adjusting the parameter of an analytical model to perform a least-square error minimization between Doppler measurements and the analytical velocity model (cf. equation 1). To ensure the well-conditioning of the problem, two other physical constraints are added: the air mass continuity equation and the water content continuity.
3.1 Doppler measurements:

The minimization of $S$ (equation 1) leads to a linear system of $M$ equations with $M$ unknowns $b_k$ $(k = 1, 2 ...M)$. $M$ is equal to nine and twenty four, respectively, for linear and quadratic models. This is equivalent to a matrix-vector equation:

$$A \cdot B = C$$

(3)

in which $B$ is the $M$-dimensional vector of the unknown parameters $b_k$, $A$ is the $M \times M$ symmetric matrix of analytical information for Doppler measurements and $C$ is the $M$-dimensional vector containing the measured radial velocities. As stated previously, the system cannot be reliably inverted without additional constraints.

3.2 The first additional constraint

This constraint is the air mass conservation equation:

$$\text{div } (\rho_0 \vec{v}) = 0$$

(4)

Where $\vec{v}$ is the air velocity in an Eulerian sense, and $\rho_0$ is the air density under the hydrostatic condition:

$$\rho(z)=\rho(z_r)\exp(-\frac{z-z_r}{H})$$

(5)

Where $z_r$ is a reference altitude and $H$ is the scale height for air pressure variations. Thus, the continuity equation (4) is rewritten in:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \frac{W}{H} = 0$$

(6)

$(U, V, W)$ are respectively the longitudinal, lateral and vertical components of the velocity vector of the wind. Each of these components is a function of the location $(x, y, z)$. This constraint is statistically verified in a least-squares sense on all measurements points and expressed as:

$$S_i = \frac{1}{N} \sum \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \frac{W}{H} \right)_{\text{analytical}}^2$$

(7)

As for the Doppler measurements, the minimization of $S_i$ leads to a linear system of $M$ equations with $M$ unknowns $b_k$:

$$A_i \cdot B = C_i = 0$$

(8)

$A_i$ is the $M \times M$ symmetric matrix of analytical information related to the air mass continuity condition, and $C_i$ is the M-dimensional vector containing experimental information.

3.3 The second additional constraint

This constraint requires the wind field satisfying the continuity equation for water ice content. The processes of condensation and evaporation are very slow compared to the time scale of radar measurements. These processes are neglected in our calculations and thus:

$$\frac{D q_i}{D t} = 0 \text{ with } q_i(Z) = a Z^b_i$$

(9)

$q_i$ is the water or ice content per unit volume sensed thought the reflectivity map $(a, b$ are experimental constants, depending of the location, altitude, temperature, etc.) and “$D q / D t$” is the derivation in Lagrangian sense. The water contents $q_i$ at the flight altitude, is mostly made of ice particles. It is derived from the measured radar reflectivity $Z$ (deduced from the zero-order moment) using an empirical relation $q(Z)$. The equation (9) can be written, in an Eulerian sense, as:

$$U \frac{\partial q_i}{\partial x} + V \frac{\partial q_i}{\partial y} + W \frac{\partial q_i}{\partial z} \frac{q_i}{H} + \frac{q_i}{H} \frac{\partial q_i}{\partial t} + \frac{q_i}{H} \frac{\partial q_i}{\partial z} \frac{q_i}{H} = 0$$

(10)

Where $v_t$ is the terminal fall-velocity of the hydrometeors (estimated from the reflectivity “$Z$”(Doviak and Zrnic, 1984)). This constraint is statistically verified in the least-squares sense and expressed as:
\[ S_2 = \frac{1}{N} \sum_i \left( U_{ai} \frac{\partial q_i}{\partial x} + V_{ai} \frac{\partial q_i}{\partial y} + W_{ai} \frac{\partial q_i}{\partial z} \right) - \left( \frac{\partial q_i}{\partial z} \frac{\partial \tilde{v}}{\partial t} + q_i \frac{\partial \tilde{v}}{\partial z} + \frac{\partial q_i}{\partial t} \frac{v_i}{H} \right)_{obs} \] \tag{11}

The minimization of \( S_2 \) leads to solve the equation:

\[ A_2 B = C_2 \] \tag{12}

\( A_2 \) is the matrix of analytical information related to the ice/water mass continuity condition, and \( C_2 \) is the vector containing experimental information.

3.4 Solution of the inverse problem with constraints

For linear and quadratic wind field assumptions, the minimization with two constraints (2) leads to a linear system of \( M \) equations with \( M \) unknowns’ parameters \( b_k (k=1,2,...,M) \) contained in the vector:

\[ \left[ \lambda A + \lambda_2 A \right] B = \left[ \lambda C + \lambda_2 C \right] \] \tag{13}

For sinusoidal wind field models, the minimization of the sum (2) does not lead to a linear system. Thus, the minimization is done using a “Levenberg-Marquart” algorithm, which is a Gradient-expansion algorithm. It combines the best features of the gradient search with the method of linearization of the fitting function (Bevington, 1969).

Three solutions are computed, corresponding to the three models. The one that provides the minimum of mean quadratic error is retained. The use of both linear and quadratic models is feasible on a real time system. The simultaneous use of the three models is more difficult due to the extra computational load required by the resolution of the nonlinear problem. Once the wind field retrieved in a reflective area, it is possible to propagate the gravity waves far away in non reflective that the aircraft foresees to cross. This subject, relative to the propagation of gravity waves, has not been addressed in this work.

4. Experimental Results

The DTCOR method was applied on a real airborne dataset collected during the international campaign IHOP 2002 (International H2O Project) over the southern Great Plains (Weckwerth et al., 2004). The main objective of IHOP was to get a characterization of the time-varying three dimensional water vapor fields and to evaluate its utility in improving the understanding and prediction of convective processes (Wakimoto et al., 2006).

The data used are from the ELDORA Radar. ELDORA/ASTRAIA (Electra Doppler Radar/Analyse Stéréoscopique par Radar Aéroporté sur Electra) was jointly built by the NCAR in Boulder and the LATMOS in France (Hildebrand et al. 1994). It is designed to provide high-resolution measurements of air motion and rainfall characteristics of very large storms.

ELDORA is a dual-beam stereoscopic side looking Doppler Radar with beams tilted fore and aft of a vertical plane normal to the fuselage and performing conical scans around an axis parallel to the longitudinal axis of the aircraft. Data from ELDORA were processed to reflect, as closely as possible, data as it would be received from a nose radar.

![Fig.3 Schematic representation of the DTCOR method applied to real data.](image)

During IHOP, the ELDORA radar was on-board the same P3 aircraft as LEANDRE II. Having both instruments together enables a direct comparison of radial velocity and reflectivity fields from the radar with the vertical measurements of water vapor mixing ratio from the lidar. Horizontal cross-section from the ELDORA radar can be obtained and also compared to data from the S-POL radar (the best observation of the bore was achieved through the north-south leg measured by the NRL P-3 between 0555 and 0617 UTC). The application of DTCOR to ELDORA data is performed as follows. The mean wind field (horizontal, vertical velocity and derivatives of the wind field) are retrieved in successive domains “\( \mathcal{D} \)” sampled during the motion of the airplane (see Fig. 3).

In this specific situation, these domains are not located directly in front of the aircraft (as for warning applications considered in the previous section) but on both sides of the aircraft since the radar in this case performed conical scanning with beams quasi perpendicular to the ELDORA track. This different configuration allows evaluating in a more drastic way the robustness of the method. Indeed the sampling chosen for civil aircraft to meet the constraints "to warn in real time" (a fast quasi-horizontal sweep at the front of the aircraft) enables a quasi-direct retrieval of the horizontal wind with a good
accuracy because its contribution on the measured radial velocities is maximal. Vertical velocities are thus essentially obtained from the physical constraints used in DTCOR. On the other hand, for the helicoidal sampling performed by ELDORA, the retrieval of the horizontal field is less direct because the horizontal component does not fully contribute to the measured radial velocities. In particular, there is the contribution of the vertical air motion and of the terminal fall velocity of the targets. Moreover, the radar scanning selected for civil aircrafts allows sweeping several times the same domain of restitution “D”. This leads to 1) increase the amount of measurements in “D” and 2) to decrease the sensibility of the retrieval to noise measurement, which in turns improves the accuracy on the temporal evolution term of the continuity equation of the tracer. Let us recall that this equation is fundamental in the present data processing because it enables the calculation of the absolute value of the vertical velocity (i.e. the constant component of vertical wind for the linear assumption), the continuity equation alone providing only the vertical variation of the vertical velocity.

This evaluation of the DTCOR method is done by comparing the retrieved parameters to those data deduced from LEANDRE II, S-POL, GLOW and the NCAR sounding (Kabeche, 2009). The retrieval domains “D” are on the order of 2.5 x 2.5 x 0.75 km$^3$ (L$x$ x L$y$ x L$z$) and taken along a meridian (y ranging from -10 to 110 km) centered at x = -3 km and z = 2.6 km MSL. These domains are taken every 1 km in the y-direction and are thus partly superimposed.

The results are plotted in Fig. 5 in function of the y-axis. The first three graphs from the top give the wind component at the center of the domain (constant terms $U_0$, $V_0$, $W_0$ of the linear model) along with the mean wind component ($U$, $V$ and $W$) in the domain “D”. The difference between the observed radial velocities measured by ELDORA and the reconstructed radial velocities from the retrieved wind is presented in panel (d). The last two graphs provide the mean reflectivity and the number of measured points used in each domain “D” by DTCOR.

The residual between the radial velocities from ELDORA and from DTCOR is shown to be +/- 0.02 m/s, providing evidence that the fit is pretty good. The horizontal wind perpendicular and parallel to the trajectory of the airplane have an
average value of 7 and 17 m/s. Therefore, it indicates an average wind of 18.4 m/s from the southwest at an altitude between 2.3 and 3 km MSL (1.6–2.3 km AGL). This wind speed is very close to the 19.3 m/s value deduced and independently from ELDORA data in a vertical plane below and along the aircraft trajectory. It is also close to the one observed by the S-pol radar or derived from the sounding at Homestead (Fig. 4(a)). The difference of altitude of the maximum wind is consistent with the one observed by the lidar GLOW.

The wind at the center of each region $U_0$ and $V_0$ is close to the mean wind except for areas where the number of points used in DTCOR is small and probably badly distributed in “D”. For instance, the horizontal winds retrieved are not so good in the southern portion of the domain (y between -110 and -90 km), where the number of measured points is less than 300. The vertical velocity shows, in the propagation direction of the bore (along the y-axis), oscillations with a spatial frequency of 11 km in the southern portion (y between -110 and -50 km), with maximum and minimum values of about -7 and -0.5 m/s. These oscillations are associated with fluctuations of U and V components indicating slightly slanted vertical motions and therefore the presence of a vertical wind shear.

The reflectivity maxima are shown to be slightly shifted from the velocity peaks in transition zones (panel (e)). This may be explained by turbulence maxima in strong wind shear transition zones. These maxima could also be interpreted as abrupt humidity change in transition zones. Thus, there is an enhancement of the radar signal (a retro-diffused signal) in transition regions from dry air to humid air in the direction of the radar beam. All these results can be compared to the other IHOP observations. They appear in good agreement with LEANDRE II (Fig. 4(b)), which identifies eight maxima/minima (waves crest) with a wavelength of 11 km, and with S-POL (Fig. 4(a)) which detects five to seven waves crest maxima/minima with a mean wavelength of 10.5 km, when the soliton approached the vicinity of Homestead. The retrieved oscillations of the vertical velocity appear also in good agreement with GLOW observations. The fluctuations of ±3 m/s of the vertical velocity, close to maximum values calculated at the vertical and below the aircraft trajectory from ELDORA measurements, are similar to those obtained from the Doppler LIDAR GLOW. On the other hand, a strong discrepancy appears for the mean vertical wind. Indeed, the fluctuations of vertical velocity appear in an overall region of subsiding air (mean velocity of -3.8 m/s) for DTCOR, in agreement with the velocities calculated from the vertically aligned beams from ELDORA (-4 m/s), but in disagreement with the analysis of the GLOW data (mean velocity of 0 m/s)(Kabeche, 2009).

Although this difference does not affect the detection of the periodic structure (for warning applications) a precise analysis of this difference is needed for the present research application.

5. Perspectives and Conclusion

The DTCOR method presented in this paper provides a 3D wind field reconstruction for research application and for real-time turbulence detection and can deal with regions of very low reflectivity around the foreseen trajectory of airborne X-band radar. Specifically, even if the data are sparse, a sufficient number of samples will allow for retrieval of the wind and turbulent structure. DTCOR is complementary to the conventional statistical approach devoted to turbulent detection. When a sufficient number of radial velocity measurements at different elevation angles are collected, the distance at which DTCOR correctly retrieves the wind field is a bit closer than when the conventional method provides a hazard warning. In contrast, at mid-range to small distances ahead of the airplane, DTCOR provides much better and more accurate predictions of turbulence hazards. The simulation conducted with synthetic data and the application on real data from the IHOP campaign showed good retrieval of the wind field. In this work only linear and quadratic representations of the wind field area were considered.

With the radar parameters used for the simulations on synthetic data (range resolution 300 m, beam-width:3°), the retrieval of turbulent structures with wavelengths greater than one kilometer is possible. With the same radar parameters, a sinusoidal wind model could give access to structures with shorter wavelengths. Another way to reduce the detectable wavelengths could be to reduce the radar cell size –depending on the available samples-, resulting in better range resolution and/or narrower lateral resolution.

References


