A Bayesian Algorithm for Tangential Deconvolution of Weather Radar Images

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1. Introduction

Application of deconvolution to weather radar images has remained a challenging problem. Deconvolution algorithms tend to amplify measurement noise, which makes straightforward inversion of the convolution process impractical. A number of algorithms have been published to improve radar image quality in the radial direction from the radar, but success in tangential deconvolution has been limited.

With traditional images such as photographs, deconvolution has been studied extensively. However, most image deconvolution algorithms cannot be applied on radar images in a straightforward manner as the algorithms are typically formulated by assuming additive noise, which is not applicable for weather radars in which the single-pulse statistical noise is multiplicative (as we shall explain below). Still, we can identify a few general principles that have led to success in various types of deconvolution algorithms. Firstly, it has been understood that the statistical model of the image noise should be accurate in order to properly understand the effect of noise on the resulting image. Secondly, the ill-posed nature of the problem should be dealt with by applying a regularization that weights the result towards more realistic images.

Although the differences between radar images and photograph-like images necessitate a somewhat different approach for radar image processing, the above mentioned principles do not present any insurmountable obstacles for radar deconvolution. For radar image noise modeling, a particular model of image noise, exponentially distributed multiplicative noise for single-pulse powers, has rather strong theoretical justifications supporting it; indeed this is more than can be said for many photograph noise models. The regularization can be approached in several ways, and there is no such strong theoretical support for any particular model, but on the other hand, the difference from photographs is perhaps smaller than in the case of noise. Thus, these approaches seem justifiable in weather radar deconvolution as well.

A general and statistically justified method to combine the noise model and the regularization is offered by Bayesian probabilistic treatment. We have developed a deconvolution algorithm for weather radar images that is based on rigorous Bayesian treatment of the properties of the image, and the process of deterioration through convolution and noise. Typically for Bayesian formulation, the *a posteriori* probability is written in terms of conditional and *a priori* probabilities. The conditional probability is formulated by requiring that the probability distribution of the multiplicative noise is the exponential distribution; this property is true for the single-pulse powers received by the radar. The single-pulse powers are required by the algorithm as input, necessitating the use of . The *a priori* probability is based on the properties of the spatial variability of rainfall, and restricts the gradient of the logarithm of pulse power, resulting in an effective "requested sharpness" parameter for the algorithm. From the Bayesian probability, a cost function is derived; this can be minimized using a standard multivariate optimization algorithm. We tested the result with radar RHI scans and observed a significant sharpening of the images.

2. Formulation of the problem

Consider an ”ideal” radar reflectivity field, which we shall call **x**. This is the image that would be seen by a radar that has a perfect pencil beam and a negligibly short pulse length. Mainly due to the statistical variation of the radar return signal and the nonzero width of the beam from the radar antenna, a real radar will see a degraded version of this image, called **y** here. In this section, we shall describe the process that forms the image **y**.

When a weather radar scans, the underlying “true” image is spatially averaged (blurred) by the square of the antenna pattern. This results in a convolution in the polar coordinates. Mathematically, the expected value of the radar pulse power y is

$$\bar{y}(\phi)=\int x(\phi-\phi')\,g(\phi')\,d\phi' \qquad (1)$$

where x is the true image, $g=f^2$ is the square of the antenna pattern f, and ϕ is the polar angle. The image is also blurred by the radar pulse shape in the radial direction, but we neglect this effect in this study. The convolution can be discretized and written in linear algebraic form as

$$\bar{y}=\mathbf{G}\mathbf{x} \qquad (2)$$

where **G** is a Toeplitz matrix (or more generally, a linear operator) which produces a convolution by *f* on a vector multiplied by it.

The actual received power is randomly determined from an exponential distribution with the given expected value. The latter effect is explained as follows: since the number of scatterers in a meteorological target is large, the central limit theorem applies such that the received real and imaginary phasor voltages *I* and *Q* are normally distributed. The power from the pulse is I^2+Q^2, and the sum of the squares of two normally distributed random variables is well known to be exponentially distributed.

The actual realized value *y* of the received power from a single pulse is a random variable with probability density function (PDF)

$$p(y)=\frac{1}{\bar{y}(\phi)}\exp\left(-\frac{y}{\bar{y}(\phi)}\right) \quad . \tag{3}$$

As this PDF only depends on the ratio of the realized value and the expected value, the statistical variation of the signal can be seen as multiplicative noise, and thus the one-pulse image formation process can be written as

$$\mathbf{y}=\mathbf{Gx}\cdot\mathbf{n} \tag{4}$$

where **n** is a sampled an exponential distribution with a rate parameter of 1.

3. Algorithm

We attempt to solve the inverse problem of restoring the ideal image **x** from the degraded image **y** by writing the probability of **x**, given **y** and some assumptions about the nature of **x**, using Bayes' theorem as.

$$p(\mathbf{x}|\mathbf{y})=\frac{p(\mathbf{y}|\mathbf{x})\,p(\mathbf{x})}{p(\mathbf{y})} \quad . \tag{5}$$

A solution that maximizes this conditional probability is called the maximum a posteriori (MAP) estimate.

The conditional probability $p(\mathbf{y}|\mathbf{x})$ can be formulated in terms of the noise. From (4), we see that

$$\frac{\mathbf{Gx}}{y}=\mathbf{n} \quad . \tag{6}$$

Since the PDF of **n** is known, we can write (for *N* discrete samples).

$$p(\mathbf{y}|\mathbf{x})\propto\prod\exp\left(-\frac{\mathbf{y}_i}{(\mathbf{Gx})_i}\right) \quad . \tag{7}$$

The problem with this definition is that the modal and expected values of the exponential distribution are very different (0 and 1, respectively, in this case), which can cause problems when using the MAP estimator and aiming to find the expected value. Also, the PDF is discontinuous at 0, which is problematic particularly for unbounded optimization methods. We can work around the problem by using the conditional distribution problem by using the conditional probability of the logarithms instead. It can be shown (we omit the proof here) that

$$p(\ln\mathbf{y}|\ln\mathbf{x})\propto\prod\exp\left(\ln\left(\frac{\mathbf{y}_i}{(\mathbf{Gx})_i}\right)-\frac{\mathbf{y}_i}{(\mathbf{Gx})_i}\right) \tag{8}$$

and this expression is adopted as our conditional probability.

For the prior $p(\mathbf{x})$, we need to assume something about the nature of **x**. It is a common assumption that meteorological targets exhibit scale-free variability statistics. This means that the logarithm of the rain rate can be modeled as a Gaussian random walk with a constant variability. Since the logarithm of the received radar power is approximately linearly related to the logarithm of the rain rate, this assumption should apply to the received power as well. With this reasoning, we construct the prior in terms of the spatial rate of change in $\ln(\mathbf{x})$, with

$$p(\ln\mathbf{x})\propto\prod\exp\left(-\frac{\mathbf{D}_r\ln(\mathbf{x})_i^2}{2\sigma_r^2}-\frac{\mathbf{D}_\phi\ln(\mathbf{x})_i^2}{2\sigma_\phi^2}\right) \tag{9}$$

where $\mathbf{D}_r$ and $\mathbf{D}_\phi$ are "differencing" matrices which give the difference of adjacent values in the radial and tangential directions, respectively. This prior also allows us to conveniently operate with $\ln(\mathbf{x})$, as with the conditional part. The parameters σ_r and σ_ϕ should be as close to the real variability of the measured target as possible. In practice, small values of σ_r and σ_ϕ lead to softer images, while large values produce sharper results but can also introduce artifacts. Thus, these can be seen as a "requested sharpness" parameters.

Our optimal estimate is the **x** that maximizes the probability $p(\mathbf{x}|\mathbf{y})$. Since we can equally validly maximize the probability of $\ln(p(\mathbf{x}|\mathbf{y}))$, the deconvolution becomes a cost function minimization problem where the cost function *E* is

$$E=\sum \ln\left(\frac{\mathbf{y}_i}{(\mathbf{Gx})_i}\right)-\frac{\mathbf{y}_i}{(\mathbf{Gx})_i}+\frac{\mathbf{D}_r \ln(\mathbf{x})_i^2}{2\sigma_r^2}+\frac{\mathbf{D}_\phi \ln(\mathbf{x})_i^2}{2\sigma_\phi^2} \quad , \tag{10}$$

which can be minimized with respect to **x** with standard optimization methods.

4. Experiments

For testing the deconvolution, we used test data from a case observed using the University of Helsinki Kumpula radar on March 19, 2011. From the original RHI scan, we selected a smaller part of 30 range gates from each of 1500 pulses, at roughly 69-75 km distance and an elevation angle of -0.4° - 8.7°. The range was convenient for the beamwidth of 1.05° to introduce a sufficient level of blurring to the meteorological features. Additionally, there was a point target (probably an airplane) in this part of the scan, which presented a good test for deconvolution.

We applied the MAP method using various parameters for σ_r and σ_ϕ . Lacking an accurate measured antenna pattern, it was assumed to be Gaussian, and various standard deviations of f were tried. The best parameters for this case were found with some experimentation to be $\sigma_r \approx 3.1$ and $\sigma_\phi \approx 0.36$, using a standard deviation of 1.05° for f.

The results of applying the MAP method are shown in Figs. 1-3. The results of using 64-sample integration to produce a "standard" radar image are also shown. The examples show that the deconvolution method seems to add a significant visual improvement to the quality of the radar image. The meteorological targets on the center right and the bottom of the images are denoised and sharpened at the edges; the widening of objects introduced by the convolution is also removed from the center right target. The object in the center of the image that was assumed to be a point target has collapsed into a much more compact and intense target. It is encouraging to note that the method works even though the signal from the point target (which constitutes only a small part of the whole image) does not follow the exponential multiplicative noise assumption. Another interesting finding was that even though the convolution only operates in one dimension, the method works better for two-dimensional images than it does for one-dimensional time series. Thus, processing the whole image at once yields better results than processing each range independently. An intuitive explanation for this is that in a 2-dimensional image, there are more neighboring pixels available from which to recover a noise-corrupted pixel by using the prior.

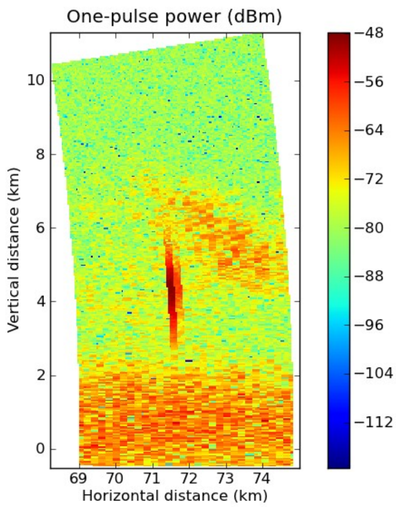

Fig. 1 Single-pulse powers from a point target and meteorological targets in an RHI scan of the UH radar. The coordinates are in kilometers, and Cartesian with the horizontal and vertical axes defined as those at the radar site (thus ignoring the curvature of the Earth).

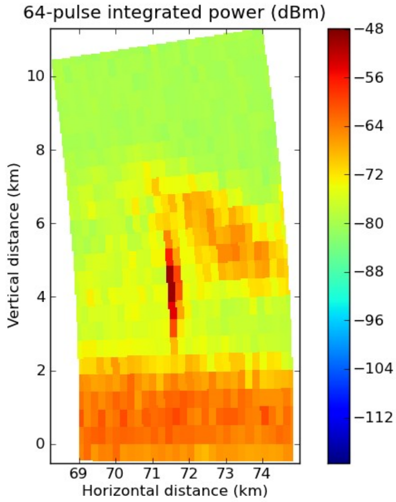

Fig. 2 The radar image of Fig. 1 after 64-pulse averaging.

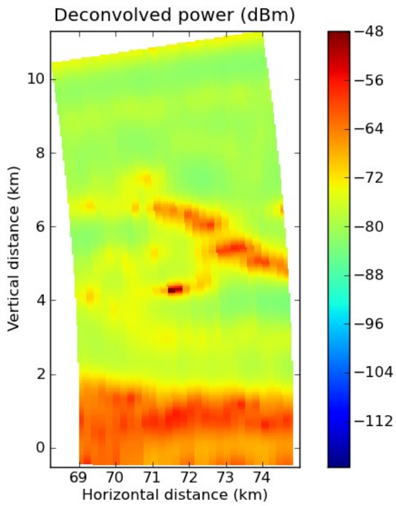

Fig. 3 The radar image of Fig. 1 after deconvolution using the MAP method.

5. Conclusion

The MAP deconvolution method seems to be able to provide a resolution enhancement that could be useful in using radar to study targets where high resolution is essential. In fact, since it operates on one-pulse power data, it can be said to combine deconvolution, denoising and superresolution methods in inverting the image degradation process. The failure of the conventional deconvolution methods to tackle the problem highlights the need for tailored algorithms for the weather radar deconvolution problem.

Finding optimal methods for determining the values of the free parameters of the prior, σ_r and σ_ϕ, is essential for making the MAP method more widely applicable. Ideally, they should be derived from a physical basis, using the horizontal and vertical rates of variation as the starting point. It should also be possible to derive the parameters as a function of the distance between adjacent samples by deriving statistical relations from large quantities of measured data. At the current state of the algorithms, however, the parameters have to be found experimentally.

The computational requirements for the MAP method are also heavy in the current implementation. Enhancing the 1500x30 point image used here takes roughly 2 minutes on a desktop computer, so it would be desirable to find an optimization method that is tailored for this problem, rather than using the general L-BFGS method. Other than that, the computation can be efficiently parallelized, which allows for code optimization that was not used in this study.

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