Kalman filtering based probabilistic nowcasting of object oriented tracked convective storms

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(Dated: 30 May 2012)

1. Introduction

Due to severe convective storms, many municipalities suffer from economic and ecological losses every year. The ability to predict the movement of the phenomenon up to a few hours ahead would provide valuable information for end-users. A standard approach for nowcasting and analyzing convective weather is so-called radar based object oriented convective storm tracking, which captures the history and movement of an individual convective storm (e.g. Dixon and Wiener 1993; Johnson et al. 1998; Handwerker 2002). However, storm tracking algorithms typically provide short term forecasts and warnings only in deterministic fashion. That is, the forecast model produces only a single forecast with limited guidance of the uncertainties related to the model. This has led to the development of the storm tracking algorithm presented in this paper, which explicitly addresses the issue of uncertainty of the forecasts.

In this study, we introduce a Kalman filter based nowcasting and tracking algorithm for convective storms. The algorithm introduces two major features in the field of convective storm tracking. First, the algorithm provides a compact way to compute smoothed estimates for various storm parameters, such as storm position and velocity, which can be further used for reliable nowcasting of the convective storm objects. Second, the estimated posterior distribution of the storm parameters can be used for deriving probabilistic nowcast products for the tracked convective storms.

2. Kalman filtering

In the field of convective storm tracking algorithms, the concept of probabilistic analysis is still rare. However, in generic engineering tracking tasks, probabilistic algorithms have been used for a long time (e.g. Bar-Shalom and Li 1995). The most widely used method applied in probabilistic target tracking is definitely Kalman filtering developed by Rudolf E. Kalman in 1960s (Kalman 1960). Probabilistic approaches, including Kalman filter, are also widely applied in numerical weather prediction (e.g. Zhou et al. 2006).

Kalman filtering can be regarded as a statistical inversion method, where the unknown state \( x \) of a system is estimated through noisy measurements \( y_1, \ldots, y_N \). When applying Kalman filtering with an object oriented convective storm tracking algorithm, each individual tracked storm is regarded as the system of interest, and the hidden state \( x \) of the system can include, for example, position, velocity and area parameters of the storm. The goal is to provide an optimal estimate of the unknown state vector \( x_k \) at \( k \)th time step using the previous noisy observations \( y_1, \ldots, y_k \) and a dynamic model for the temporal development of the state parameters. In case of the storm tracking, the observation vector \( y_k \) include variables that can be directly measured from the radar-detected storm object, such as uncertain measurements of the area and centroid position of the storm.

Kalman filtering assumes that the system can be presented with a linear stochastic state-space model with Gaussian noises, formulated as

\[
\begin{align*}
    x_k &= F_k x_{k-1} + w_{k-1} \\
    y_k &= H_k x_k + v_k
\end{align*}
\]

where \( F_k \) is the state transition model, \( H_k \) is the observation model and \( v_k \) and \( w_{k-1} \) are additional noise variables. The state transition model \( F_k \) defines how the model state evolves between successive time steps. However, no deterministic model describes any system perfectly and therefore the random variable \( w_{k-1} \) is added to describe uncertainties of the modeled system. Thus, \( w_{k-1} \) is also called as the process noise and it is assumed to follow Gaussian distribution with \( w_{k-1} \sim \mathcal{N}(0, Q_{k-1}) \). The observation model \( H_k \), in turn, defines how the model state is mapped from the state variables to the observations. The random vector \( v_k \) is the measurement noise, which relates to the uncertainty of the measurements \( y_k \). Like the process noise, it is assumed to follow Gaussian distribution with \( v_k \sim \mathcal{N}(0, R_{k-1}) \). It is also assumed that both \( v_k \) and \( w_{k-1} \) are white noise processes and independent on the initial state \( x_0 \).

The Kalman filter estimate \( \hat{x}_k \) and its covariance \( P_k \) for the hidden state \( x_k \) at \( k \)th time step are estimated recursively with the following two-phase algorithm.

\[
\begin{align*}
    \hat{x}_k &= F_k \hat{x}_{k-1} + K_k (y_k - H_k \hat{x}_{k-1}) \\
    P_k &= (I - K_k H_k) P_{k-1}
\end{align*}
\]

where \( K_k = P_{k-1} H_k^T (H_k P_{k-1} H_k^T + R_k)^{-1} \) is the Kalman gain.
• Prediction step:

\[ m_k^* = F_{k-1} m_{k-1} \]
\[ P_k^* = F_{k-1} P_{k-1} F_{k-1}^T + Q_{k-1}. \]

• Update step:

\[ K_k = P_k^* H_k^T (H_k P_k^* H_k^T + R_k)^{-1} \]
\[ m_k = m_k^* + K_k (y_k - H_k m_k^*) \]
\[ P_k = (I - K_k H_k) P_k^*. \]

The variables \( m_k^* \) and \( P_k^* \) in the prediction step denote the predicted state estimate and covariance, that is, the estimated state estimate and covariance before the they are updated with an evidence given by the measurement. In the update step, the state estimate and covariance are further modified using the noisy measurement \( y_k \). In (3), \( K_k \) is Kalman gain, which defines the weight given to the observation with respect to predicted state. Furthermore, it is assumed that at the first time step the system state has an initial state \( m_0 \) and covariance \( P_0 \). From Bayesian point of view, the estimates \( m_k \) and \( P_k \) can be regarded as the mean and covariance of the posterior distribution of the state \( x_k \), given the noisy measurement \( y_k \) and the prior mean \( m_k^* \) and covariance \( P_k^* \). For further discussion on general theory of Kalman filtering, we refer to various sources in the literature (e.g. Bar-Shalom and Li 1995; Simon 2006).

3. Kalman filtering model for tracking convective storms

3.1 Applied Kalman filtering model

Kalman filtering model for tracked convective storms includes state estimation of variables of storm cells. In here, the initial tracking, i.e. storm identification and assignment, are performed with the clustering based tracking algorithm by Rossi and Mäkelä (2008). Thereafter, we apply Kalman filtering to the tracked storms, which improves the state estimation of the storm cells. The estimated state information can be used, for example, in the storm nowcasting.

The initial storm tracking algorithm applies the radar reflectivity factor of 35 dBZ for identifying storm areas, which enables tracking of full storm systems, such as multicellular storms or MCS (Dixon and Wiener 1993). The identified storm areas are further processed using the morphological closing operation with a round structuring element with 3 km diameter (e.g. Gonzalez and Woods 2008).

In many object oriented storm tracking applications the nowcasting of the storm cells is based on linear extrapolation of the position using previous position or velocity estimates (e.g. Dixon and Wiener 1993; Handwerker 2002). Therefore, also the state variables of our model are the centroid position \((x,y)\) of an identified storm and corresponding velocity components \((\dot{x}, \dot{y})\), denoted as a state-vector \( x = (x, \dot{x}, y, \dot{y})^T \). The only measured variables are the centroid position coordinates of the identified storms and therefore the elements corresponding to the velocity observations in the measurement matrix \( H_k \) are zero. The velocity information is estimated later on using the applied dynamic model and the Kalman filtering equations. Thus, the linear observation model \( H_k \) of the system is

\[ H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \]

and the measurement noise \( v_k \) is zero mean Gaussian white noise with covariance

\[ R_k = \begin{bmatrix} r^2 & 0 \\ 0 & r^2 \end{bmatrix}. \]

In (5), the parameter \( r \) [length] describes the noise intensity of the measured centroid position. The uncertainty comes from various sources, such as attenuation, randomness of the reflectivity measurements and resolution limitations defined by the radar contribution volume, which results in random deformation of the identified storm shape and consequently in fluctuations in the centroid position. The measurement noise deviation of both \( x \) and \( y \) is of the order of \( r \), which helps us to choose a suitable parameter for \( r \). However, it is difficult to estimate the contribution of each noise source objectively and therefore in this work the noise parameter \( r \) is selected intuitively. Moreover, the radar-detected centroid is only a noisy measurement of the centroid of the true storm cell, which includes not only the intense precipitating part observed by the radar, but also the non-measurable updraft and downdraft of air mass.

In addition to the measurements and the measurement model, Kalman filtering requires a reasonable dynamic model to estimate the state variables of the tracked storms. Here, we adopt the continuous white noise acceleration model, where position and velocity coordinates of a target are modeled with a linear state-space model and the acceleration is assumed to be a zero mean Gaussian random process (e.g. Bar-Shalom and Li 1995). This means that the velocity of the target stays...
unchanged on average, but over each time step, it undergoes a small Gaussian random change. The state transition model \( F_t \) and the process noise covariance \( Q_t \) of the discretized white noise acceleration model are (see e.g. Bar-Shalom and Li 1995 for a detailed derivation)

\[
F_t = F(\Delta t) = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad Q_t = Q(\Delta t) = q \begin{bmatrix} \frac{1}{3} \Delta t^3 & \frac{1}{2} \Delta t^2 & 0 & 0 \\ \frac{1}{2} \Delta t^2 & \Delta t & 0 & 0 \\ 0 & 0 & \frac{1}{3} \Delta t^3 & \frac{1}{2} \Delta t^2 \\ 0 & 0 & \frac{1}{2} \Delta t^2 & \Delta t \end{bmatrix}, \quad (6)
\]

where \( \Delta t \) is the interval between two time steps and \( q \) [length\(^2\) (time step)\(^3\)] defines the process noise intensity. Random changes in the velocity over the time step length \( \Delta t \) are of the order of \( \sqrt{q \Delta t} \), which helps us to choose a reasonable parameter for the process noise. Like with the measurement noise, the value of \( q \) can be chosen subjectively, based on an educated guess or expert knowledge on the storm velocity behavior.

### 3.2 Nowcasting with Kalman filter

One of the main applications of the Kalman filter derived position and velocity is naturally short term prediction of the storm cells. This can be done by estimating the state parameters with the Kalman filtering prediction step (2) extended to the forecast time. Thus, the predicted state mean and covariance on the \( k \)th step with the lead time of \( t_f \) are

\[
m_{k+t_f}^\ast = F(t_f) m_k, \quad P_{k+t_f}^\ast = F(t_f) s P_{t_f} \left( F(t_f) \right)^T + Q(t_f), \quad (7)
\]

where \( F(t_f) \) and \( Q(t_f) \) are defined as in (6). As illustrated below in Section 5, an optional parameter \( s > 1 \) can be used to compensate the overestimation of the probability forecasts in case of decaying or transforming storm cells. It can be also shown that if the scaling is included in the prediction step of the Kalman filter (2), it corresponds to the Kalman filter with fading memory, where more weight is given to the recent observations with respect to the older observations (e.g. Simon 2006). The final nowcast for the storm cell is obtained by placing the initial storm to the location of the estimated centroid position.

### 3.3 Dealing with splitting and merging

Important special cases in the convective storm tracking are the storm splitting and merging. A storm can split into two or more new storms, or a cluster of storms can merge to the same storm. Although the initial storm tracking detects splits and mergers, they are a complicated in the filtering, since the original Kalman filter is not designed to deal with such a problem.

In here, we propose the following heuristic approach for dealing with the splitting and the merging. In case of the merging, the predicted state vectors and covariance matrices of the merging storms are simply combined with the area-weighted average of the merging storms. If a storm splits, each split successor storm inherits velocity components from the predicted state estimate of the predecessor storm. However, the centroid position estimates are taken directly from the measured centroid positions of the split storms. Each split storm cell also inherits the state covariance of the predecessor cell.

### 4. Ensemble nowcasting of convective storms

In addition to the estimated state parameters, Kalman filter provides the covariance matrix of the state parameters. Assuming that the state vector follows a multidimensional Gaussian distribution, the state estimate and its covariance uniquely define the probability density function of the state. This probability density function can be used for deriving different probabilistic nowcasting products for the tracked convective storms, which is the main motivation of this paper.

Here, we estimate the thunderstorm occurrence probability by generating ensembles with forecasted state mean \( m_{k+t_f} \) and covariance \( P_{k+t_f} \). The ensembles are generated in the following way. First, the forecasted state mean \( m_{k+t_f} \) and covariance \( P_{k+t_f} \) are calculated with (7). Second, we draw a random sample from the distribution defined by \( m_{k+t_f} \) and \( P_{k+t_f} \). Third, the forecasted storm at time \( k+t_f \) is generated by placing the initial storm to the position coordinates of the random sample.

To ease the ensemble production, the identified storms are approximated with ellipses fitted on the identified cell. The ellipse fitting is performed through the principal component analysis (see e.g. Jolliffe 2001); a covariance matrix \( C \) is estimated for the boundary points of the identified cell, after which we obtain ellipse main axes by calculating the principal component vectors of the covariance matrix. The length of each principal component vector equals to the variance of the cell boundary points along the corresponding principal axis. Here, the length of each ellipse axis is chosen such that it is two times the corresponding principal component.
Dance et al. (2010) introduced an analogous method for deriving strike probability nowcasts for tracked storm objects, i.e. the probability that a given location will be influenced by a storm within the forecast time period, using constant predefined deviation parameters for the storm velocity. In our work, both the speed and the location uncertainties are estimated recursively through Kalman filter. Moreover, the uncertainty depends on the length of the history; the covariance matrix is larger with storms having shorter history and it converges toward a steady-state covariance as more evidence of the storm movement becomes available.

5. Case examples and algorithm evaluations

The algorithm was tested with data obtained from Finnish Meteorological Institute’s eight Doppler C-band weather radars covering almost the whole Finland. Approximate constant altitude PPI images (pseudo-CAPPI) of 500 m altitude with 5 min temporal and 1 x 1 km spatial resolution were applied for the detection of storm objects.

For the overall look, Figure 1 shows an example of the Kalman filtering tracking on Aug 9 2005, where white polygons depict tracked storm objects. Kalman filtered track positions (green lines) are smoother than the non-filtered storm positions (red lines), resulting in a more consistent track. Stable, smooth behavior of the track improves the visualization of the storm tracks in end-user products. Moreover, the Kalman filter based velocity estimates are consistent with the smoothed track history, although the tracked storms split and merge frequently. Figure 1 shows also the development of the forecasted position covariance (upper right image). The dashed circle shows the area, in which the forecasted centroid falls with the probability of 95%. As illustrated below, this uncertainty can be used for realistic storm probability forecasting. Lower right image of Figure 1 shows an example of storm occurrence probability forecasting, where probability forecasts of lead times 20, 40 and 60 minutes are overlaid on each other.

In order to verify the nowcasting capabilities of the algorithm, it is necessary to evaluate the performance using well-established skill metrics. In this work, we applied commonly used probability detection (POD), false-alarm ratio (FAR) and critical success index (CSI) (e.g. AMS glossary of meteorology 2012). The verification was performed using data from three cases: August 9 2005, August 14 2007 and June 1 2010. Overall 1261 complex storm tracks, i.e. tracks with splits and mergers (Dixon and Wiener 1993), were identified in the initial tracking. For the POD, FAR and CSI calculation, the resolution was downgraded to 5 by 5 km. A grid pixel was considered active if any point within the grid pixel was influenced by an identified storm. Consequently, the forecast was considered successful, if both forecasted and true grid pixels were active. A false-alarm was assumed, if the forecasted grid pixel was active, but the true grid pixel was inactive. A failure occurred, if the true grid pixel was active, but the forecasted grid pixel was inactive.

Figure 2 shows POD, FAR and CSI values of the track-by-track analysis using the parameter settings $r = 2 \text{ km}, \sqrt{\Delta t} = 3.8 \text{ km/h}$. In the track-by-track analysis, we calculate skill scores only against storms, for which the history exceeds the lead time (Dixon and Wiener 1993). This is reasonable as the storm tracking algorithms are developed to track and forecast movement of existing storms, and they are not able deal with the temporal evolution or anticipate developing convection. Thus, the track-by-track analysis measures the nowcasting capability of the algorithm along the track. The presented results are competent with forecasting methodologies introduced with other storm tracking algorithms. For example, Dixon and Wiener (1993) calculated similar track-by-track results with TITAN. In terms of CSI, the results calculated with our algorithm are slightly better. However, it is important to note that the data set used for verification is different in this study.
The probability algorithm was also validated. In order to provide realistic probability forecasts, the estimated probabilities should be in accordance with the true observed frequencies of the forecasts. Therefore, we estimated the observed frequencies for lead times 20-60 minutes with the following procedure. The predicted probabilities were first rounded to the closest integer. Thereafter, the predicted probability fields were compared against the true observed storm cells. Since the probability forecasts were created by predicting the storms approximated with ellipses, also the observed storm were approximated with the same ellipse fitting procedure (see Section 5). For each discrete probability value ranging between 1 - 95 %, both the number of pixels under the area of the storms and the total number of pixels having the same probability value were counted. With a given probability value, the proportion of the pixels under the influence of the storms to the total number of pixels estimates the observed frequency.

Figure 3.a shows the reliability diagrams for the probability values 1-95 %, where predicted probabilities are plotted against the observed frequencies. The correspondence between the predicted probabilities and the observed frequencies is good until 50 %, after which the predicted probabilities start to overestimate the true probabilities. With shorter lead times, the correspondence is better. The overestimation of the large probability values is expected, since the algorithm does not explicitly deal with growth or decay of the storms. Individual convective cells have typically a life time less than 30 minutes, and therefore many storms decay or undergo a significant deformation during the forecast time, which is not estimated by the algorithm. However, the additional uncertainty caused by the growth and decay effects can be compensated by increasing the factor $s$ in (7). Figure 3.b presents the reliability diagrams forecasted with $s = 2$. The overestimation with the large probability values is reduced. On the other, large probabilities are estimated with fewer pixels, making the estimation less reliable. Still, the additional scaling of the covariance matrix seems to improve the overall reliability.

In the final nowcasting application the probability can be further calibrated according to the estimated reliability diagram. However, since maximum observed frequency is around 80 %, also the maximum risk in the calibrated forecasts cannot exceed this value.
6. Discussion and future directions

This work presented a Kalman filtering based algorithm for tracking convective storms. The algorithm applies the white noise acceleration model to produce a smoothed and consistent estimate for the position and velocity of a storm. When applied to the storm nowcasting, the algorithm produces competent performance results in comparison to other tracking methods. In addition, the filter can predict storm occurrence in conjunction with associated uncertainties of the forecast. These risk forecasts are consistent with observed frequencies, verifying that the proposed methodology reflects the true uncertainties of the forecasts.

Kalman filtering provides a well-established and compact way to fuse the estimation of various storm parameters in the same algorithm. The model presented in this paper includes only storm centroid and velocity components, but it can be extended also with other state variables, such as storm area or echo top altitude, if their temporal behavior and relationship with other parameters can be formulated with a stochastic state-space presentation. Moreover, the noise parameters of the Kalman filter have physical quantities, which help us to choose reasonable parameterization for the filter. In this paper, the noise parameters are selected heuristically, but if the noise behavior of the storm parameter can be estimated objectively, they can be directly included in the model. The Kalman filtering based storm tracking is also a step towards adaptive tracking, since the parameters of the filter can be changed according to the varying noise levels. For example, in the case of attenuation, beam blocking etc., the measurement noise covariance of the filter could have increased values. In addition, of different storm systems, like mesoscale convective systems or supercell storms, can behave differently. Therefore, also the process noise could be tuned adaptively in different storms.

Kalman filter provides a convenient way to combine uncertainty contributions from various sources. Therefore, the filtering algorithm presented in this paper could utilize data from other data sources. As an example, lightning data provides information on the location and movement of individual storms. This information could be incorporated as one of the state variables for the storm system.

Acknowledgements

This study has been supported by the Finnish Funding Agency for Technology and Innovation (Tekes) within Heavy Rainfall Processes (RAVAKE) project, Väisälä Foundation and Finnish Society of Automation. The participation of V.Chandrasekar in this research is supported by the National Science Foundation and the Tekes FiDiPro program.

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