Simulation of the Radar Cross-Section of Wake Vortices in clear air

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Introduction

- Real-time sensing of Wake Vortices at take-off and landing for reducing aircraft separation
- UCL is developing a software for the calculation of the radar cross section of wake vortices in the framework of SESAR P12.2.2 project, with Thales Air Systems
- Simulation in two steps
  - Evolution of pressure, temperature and humidity versus time for the calculation of the refractive index
  - Calculation of the Radar Cross Section (RCS)

Simulation of the refractive index of the wake vortex

- Refractive index of air (Thayer)

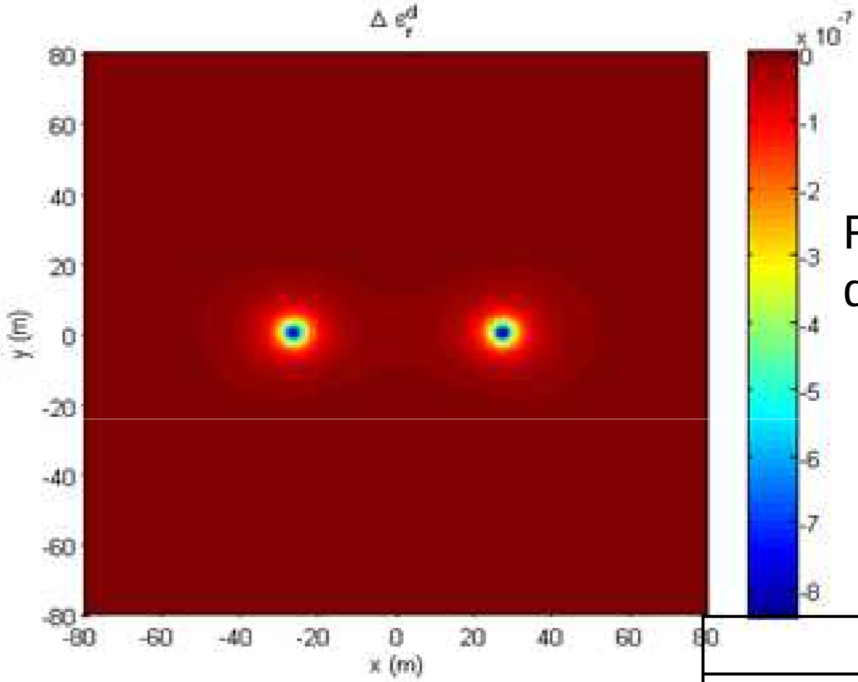
$$N = 0.776\frac{p_d}{T} + 1.33\frac{p_{CO2}}{T} + 0.648\frac{e}{T} + 3.77610^3\frac{e}{T^2}$$

- Various mechanisms responsible for the variation of the refractive index
  - the contribution of the propulsion from the reactors
    - linked to the turbo reactor stream
    - seems to influence the refractive index in the region close to the aircraft due to high temperature.
    - not considered in this preliminary model because we are only interested in the vortices a few wingspan away from the airplane.

Simulation of the refractive index of the wake vortex

- The radial density gradient in the vortex cores
  - lower pressure in the core
  - mainly depends on the airplane type that influences the intensity of the flow
- The adiabatic compression of the fluid surrounding the core
  - The transport of the atmospheric fluid in the oval surrounding the vortices
  - transports the air from one place to another, assuming adiabatic compression when the oval descents and the ambient pressure increases

Simulation of the refractive index of the wake vortex



Relative dielectric constant variation due to density 5s after roll up

Parameters	Values
Airplane mass	M=250000 Kg
Wingspan	B=68m
Airplane velocity	V =133m/s
Ambient pressure	$p_a\ = 100000\ pa$
Ambient absolute temperature	$T_a\ = 288\ K$
Water vapor content gradient	$m_q\ = -8 \times 10^{-8}$ Kg/mKg

Simulation of the refractive index of the wake vortex

- Parameters used for the simulation

$$V_0 \cong \frac{\Gamma_0}{2\pi b_0} \;;\; \Gamma_0 = \frac{Mg}{\rho V b_0}$$

$$t_0 = \frac{b_0}{V_0} \;;\; \tau = \frac{t}{t_0}$$

$$N^2 = \frac{g}{\theta_0}\frac{d\theta_a}{dz}$$

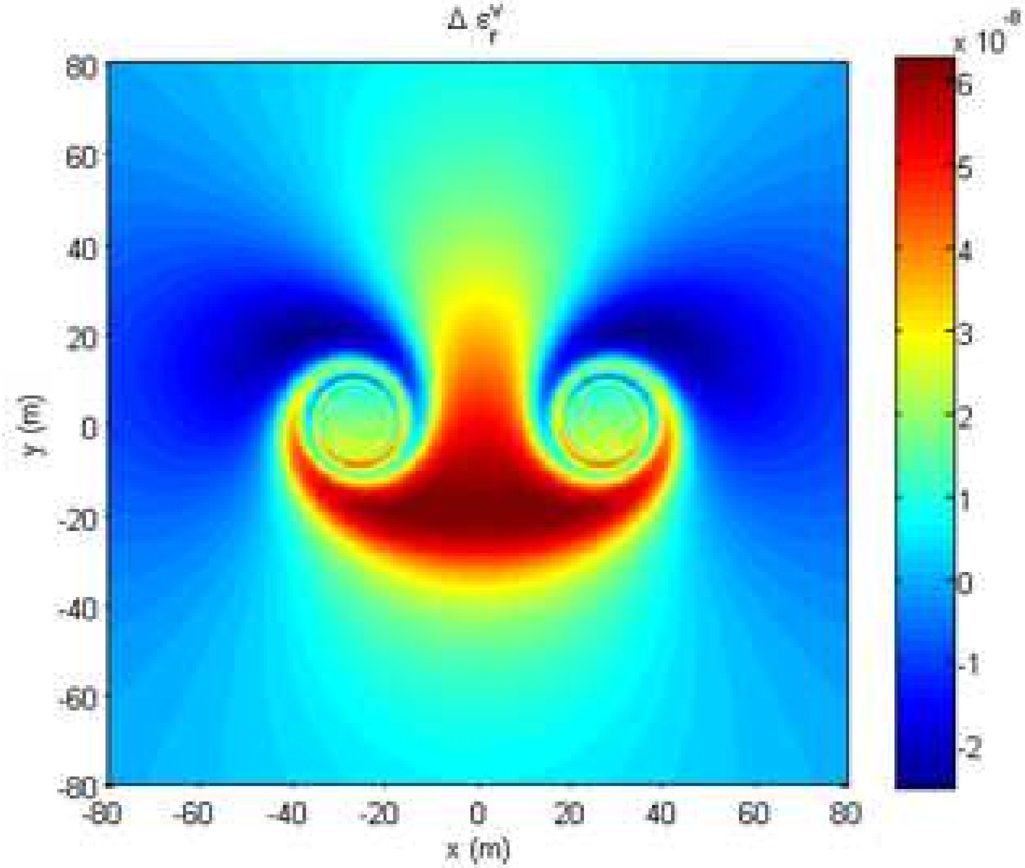
  - Γ_0 is the root velocity,
  - V_0 the descent velocity of the vortex pair,
  - b_0 the initial vortex spacing, B the wingspan,
  - V and M are the aircraft velocity and mass,
  - ρ is the air density, g the gravity constant, t_0 is the characteristic time

Simulation of the refractive index of the wake vortex

- Use of incompressible Navier-Stokes equation, with the Boussinesq approximation, describing the movement.
- The water vapor concentration obeys to the convection-diffusion equation (passive tracer “carried away” by the fluid)
- Equations solved by 2D pseudo-spectral numerical methods (all the fields have to be periodical, some mathematical artifacts are used to represent periodic pressure, water vapor, velocity and potential temperature)
- Runge-Kutta method of order 3
- hyper-viscous term in the equations for the dissipation of the high wavenumbers

Simulation of the refractive index of the wake vortex

- Dielectric permittivity variation due to water vapour, 20s after roll up of the vortex



Calculation of the radar backscattering of wake vortices

- Calculation of the radar cross section

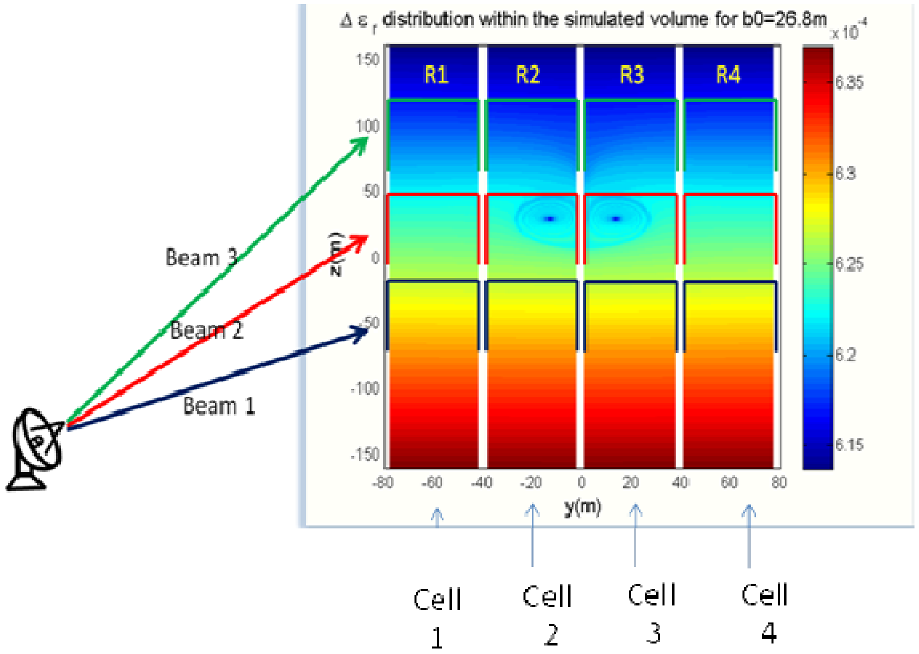
$$P_r(\overline{x_r}) = \frac{\omega \varepsilon_0 k_0^2 A^2}{32\pi^2 |\overline{x_r}|^4} |I^2|$$

$$I = \int_V \Delta\varepsilon_r f(x,y,z) e^{-2jk_0\xi} dV$$

$$RCS = \frac{k_0^2}{4\pi} |I|^2$$

  - ξ is the distance between the receiver and the volume element dV,
  - k_0 is the incident wavenumber,
  - f(x,y,z) is the radiation pattern,
  - A is the amplitude of the incident field

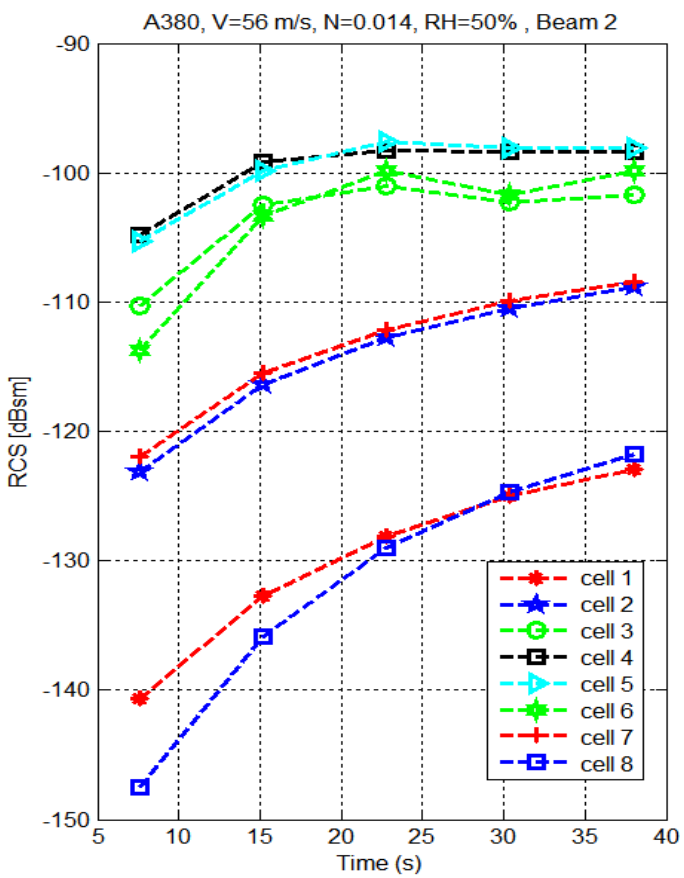
Calculation of the radar backscattering of wake vortices

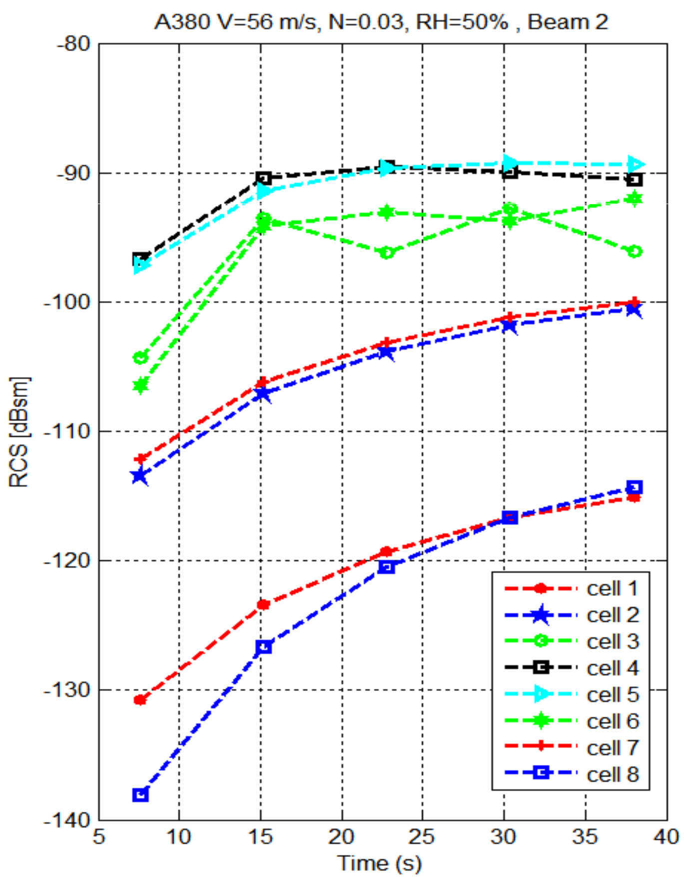


- 2D permittivity plane repeated in x direction, with no variation
- Oscillatory integral I
  - integration method proposed by Li : replaces the integral by the resolution of a system of differential equations
  - More precise than the method of Shariff and Wray

Preliminary results

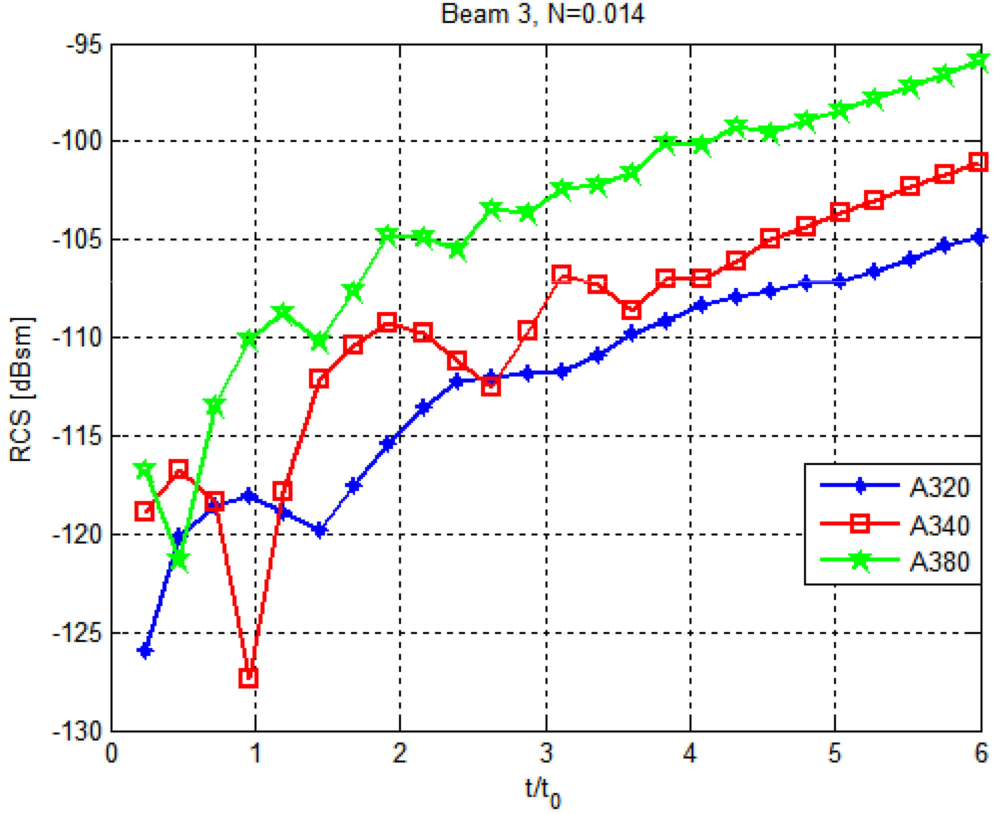
- RCS simulated for a A380 airplane, flying at 100m, with a velocity of 56 m/s, in a stratified air, with 50% relative humidity and two different Brunt-Väisälä frequencies





Preliminary results

- Radar cross section variation versus vortex evolution time



Conclusion

- Radar cross sections are calculated
  - for a wide frequency range,
  - various airplanes
  - various tropospheric parameters

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Thank you for your attention