DATA ASSIMILATION: TARGETED OBSERVATIONS AND PARAMETER ESTIMATION

ASU SCHOOL OF MATHEMATICAL & STATISTICAL SCIENCES
ARIZONA STATE UNIVERSITY

THOMAS BELLSKY, ERIC KOSTELICH, & ALEX MAHALOV
Arizona State University

MCRN www.math.climate.org

Research Summary: Data Assimilation (DA)

1. **Targeting observations in DA**
   - Strategy locates observations at points of highest ensemble variance to minimize state and forecast errors.
2. **Convergence of DA scheme**
   - Determining convergence rates for targeting strategy versus randomly located observations.
   - Vary fixed quantities to determine effect on convergence of DA scheme.
3. **Estimating model parameters with DA**
   - Accurately tuning fixed parameters with novel method.

Two Model Problems

1. Front solution to Burgers' equation

- Estimating an exact non-dissipative, traveling wave front solution to Burgers' equation:

$$u_t + uu_x = \nu u_{xx}.$$

- Random initial ensembles: Brownian motion.

2. Lorenz-96

- Toy model of a theoretical weather state at $N = 40$ points on a latitude circle with cyclic boundary conditions:

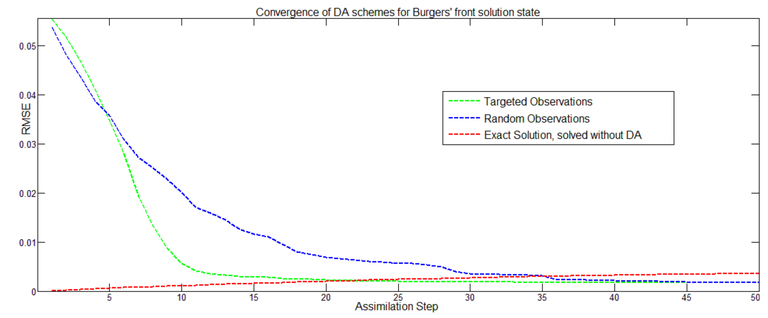
$$\frac{\partial x_j}{\partial t} = (x_{j+1} + x_{j-2})\, x_{j-1} - x_j + F,$$
$$x_{j \pm N} = x_j.$$

- Nonlinearity simulates advection and conserves energy.
- Linear terms dissipate the total energy.
- F represents external forcing; strongly determines chaotic properties of the model.

Local Ensemble Transform Kalman Filter (LETKF)

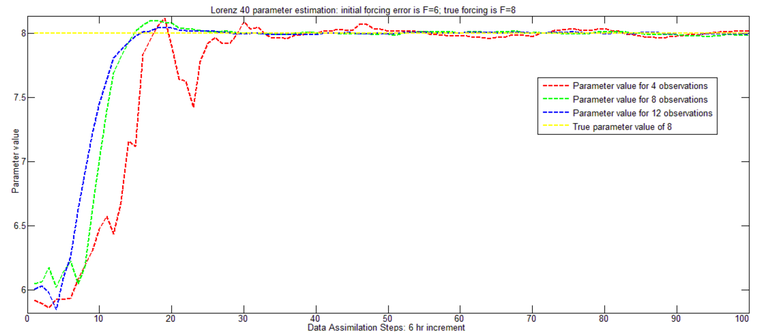
- Time efficient, accurate ensemble DA method.
- Low-rank filter capturing current covariance model error.
- Localizes analysis about each state location.
  1. Reduce spurious correlations for distant positions.
  2. Combined analysis explores larger dimensional space.
- Use with targeting strategy and parameter estimation.
- **Method: Vary localization window, number of observations, and ensemble size to determine effect these quantities have on the convergence of LETKF.**

Results: Targeting Observations

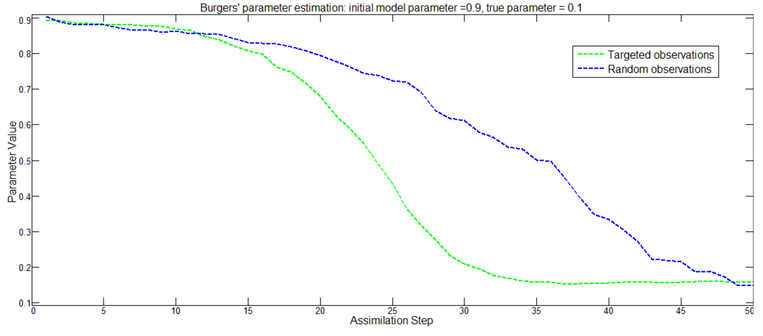


- **Largest ensemble variance targeting strategy 'skillful' in estimating/forecasting state for both models.**
- Def (Lorenz): **Skillful**: Method beats random locations.
- **Doubling number of observations or size of localization window halves the DA convergence time for both Burgers' and Lorenz-96 with small forcing ($F < 2$).**
  1. **Conjecture**: For non-chaotic systems, these halved rates of convergence hold.
  2. **Conjecture**: Rates of convergence scale to the magnitude of chaoticity for Lorenz-96 with larger forcings.

Results: Parameter Estimation



- Novel method: **LETKF+sEnKF**.
- LETKF as usual to update the analysis state.
- Separately use a non-localized ensemble Kalman filter (EnKF) on augmented ensemble of states (x, p).
- Above p is a vector of forecasted parameter values.
- Estimating Burgers' diffusion term, forcing in Lorenz-96.
- LETKF+sEnKF parameter estimation is successful for both models.



Future Work

Construct rigorous theorems suggested by these results.

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