Methods of optimal planning of remote sensing experiment in problems of satellite meteorology.

Nicolai Uvarov[1], Anton Sokolov[2], Anatoly Chavro[3].

[1]*G. V. Kurdyumov Institute for Metal Physics, NASU, Ukraine,* 03680, Kiev, Academician Vernadsky blvd. 36. [2]*Laboratoire de PhysicoChimie de l'Atmosphère (CNRS EA 4493), Univ. du Littoral Côte d'Opale & Univ. Lille Nord de France, 189A Av. Maurice Schumann - 59140 Dunkerque*

[2]*Institute of Numerical Mathematics RAS, Russia, 119991, Moscow, ul. Gubkina, 8.* *mail to: uvarovn@gmail.com

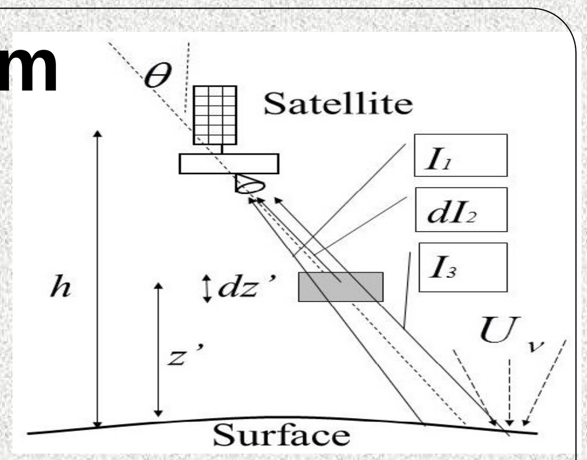
While optimum planning of satellite experiment it is necessary to take into account the physical nature of measured values. The idea of an increase of the spectral resolution to obtain the high precision of retrieval leads to decreasing of the signal/noise ratio in such "narrow" channels. The alternative idea is an "optimal" merging of radiative energy in the various (correlated) spectral ranges in a number of "superchannels". As its was shown[1], such methods can exceed a "complete" experiment of high resolution in an information content.

In this work the techniques of an optimum choice are considered of spectral channels with the fixed and variable widths. The following methods of the optimization and optimal planning were employed for a remote sensing satellites experiments: DRM[2](analysis of Data Resolution Matrix), DRM(SVD)[3], Jacobians, Iterations (selection of the satellite channels is defined by Entropy Reduction)[4], superchannel technique (spectral channel with variable width - based on maximizing determinant of Fisher's information matrix)[5].

The "best linear estimate" method and the "variational" technique were employed for the inversion of measurement data by the channels, obtained by various selection techniques. The atmospheric temperature and humidity profiles was retrieved.

The retrieval error was calculated for the data of different spectral resolution for those channel selection techniques.

Instrument simulation – forward problem



$$y_{\nu_i\mu_j} = \int_0^\infty d\nu \int_0^1 \xi_{\nu_i}(\nu) g_{\mu_j}(\mu) I_\nu(h,\mu) d\mu + \gamma_\nu$$

-signal measured at frequency *v*, at the angle of θ where: $\mu = \cos\theta$;

$\xi_{\nu_i}(\nu)$ - spectral response function; $g_{\mu_j}(\mu)$ - angular function; γ_ν - instrument error.

Monochromatic radiation:

$$I_\nu(h,\mu>0) = I_1 + I_2 + I_3 = \varepsilon_\nu(\mu) B_\nu(T_0) \exp\left(-\frac{1}{\mu}\int_0^h k_\nu(z'')dz''\right) + \int_0^h \frac{k_\nu}{\mu} B_\nu(z') \exp\left(-\frac{1}{\mu}\int_{z'}^h k_\nu(z'')dz''\right) dz' + a_\nu(\mu) \exp\left(-\frac{1}{\mu}\int_0^h k_\nu(z'')dz''\right) U_\nu$$

$k_\nu(T(z), c_i(z))$ - absorption (LBL algorithm); $B_\nu(T(z))$ - Plank function; $\varepsilon_\nu(\mu)$ - emissivity; $a_\nu(\mu)$ - reflection; $\varepsilon_\nu(\mu) + a_\nu(\mu) = 1$

Atmospheric parameters retrieval – inverse problem

$x = (T_0, \vec{T}, \vec{q})$ - sea surface temperature, atmospheric temperature and humidity profiles

- **Linear:** $R: \hat{x} = Ry$

$R_{LSR} = S_x A^* (A S_x A^* + \Sigma_\nu)^{-1}$

A - Jacobian

Σ_ε - error covariance

S_x - predictor x covariance

- **Non-linear (variational). Minimization of the cost function:**

x - vector to retrieve, x_a - a priori estimate, y_m - measurement, $y(x)$ - forward model, S_a, S_m - weight matrixes.

$$J(x) = \frac{1}{2}\left\{(x_a - x)^T S_a^{-1} (x_a - x) + (y_m - y(x))^T S_m^{-1} (y_m - y(x))\right\}$$

- **3000 samples were taken from the ECMWF Databank for calibration and 300 for verification, the satellite measurements were simulated.**
- **Linear Reduction estimate is taken as the a priori estimate for variational technics.**
- ***x*** **after normalization was projected at the PC(EOF) subspace.**
- **Jacobean was calculated analytically for the modified Newtonian minimization.**

Methods of Chanel selection

1.DRM

Values of the diagonal elements of a DRM matrix is a criteria of a channel usefulness.

$$\hat{x} = R_{LSR} \cdot y$$

$$\hat{y} = A \cdot \hat{x} = A \cdot R \cdot y \equiv DRM \cdot y$$

$$\max_{ii}(DRM_{ii})$$

2.SVD(DRM)

$$SVD\left(\Sigma^{-1/2} \cdot A \cdot S_x^{1/2}\right) = U \Lambda V^T$$

Cut nosy components:

$$\Lambda^2 \equiv \sigma_b^2 / \sigma_0^2 \le 1/9 (\approx 10\%)$$

$$B_{\hat{x}}^{-1} = B^{-1} + A^T \Sigma^{-1} A \Rightarrow 1/\sigma_a^2 \approx 1/\sigma_b^2 + 1/\sigma_0^2$$

$$\tilde{A}: U_p \Lambda_p V_p^T \qquad \max_{ii}(\tilde{DRM}_{ii})$$

3.Jacobeans

$$J = \Sigma^{-1/2} \cdot A \cdot S_x^{1/2}$$

h - *line of the Jacobean which corresponds to the given channel (weight function)*

Channel selection criteria:
line h is selected that has a maximum of the ratio of the amplitude of the peak to the half width at half maximum.

4.Iterative method

At each iteration the a posteriori covariance matrix is update: $\tilde{S}_{\hat{x}}^{-1} = S_x^{-1} + hh^T$

The criteria of a channel usefulness is an Entropy Reduction (ER): $ER_i = -\frac{1}{2}\log_2\left(\det\left((\tilde{S}_{\hat{x}})_i \cdot (S_x)^{-1}\right)\right)$

5. Channel merging – Superchannel technique

First step – spectral information analysis:

We resolve eginvalues problem for spectral covariance matrix $Cov(y) = J \cdot J^T$ *to obtain the number of available independent information components in spectra:* $M: \{\lambda_i^* > 1\}_{i=1}^M$; $J = \Sigma^{-1/2} \cdot A \cdot S_x^{1/2}$

λ *- eigenvalues of the spectral covariance matrix;* $U_{N\times M}$ *- eigenvectors.*

Second step – spectral intervals merging:

We introduces new "superchanels", which is obtained by spectral intervals merging:

$$Y_M = P_{M\times N} \cdot y_N + \gamma_M$$

The elements p_{ji} *of matrix* $P_{M\times N}$ *is ether 0 or 1, shows, whether include interval j to superchanel i.*

Matrix $P_{M\times N}$ *is defined from:* $\det[G(P_{M\times N})] \Rightarrow \max$

$$G = (P_{M\times N} \cdot U_{N\times M})^T (P_{M\times N} \cdot U_{N\times M}) \equiv K^T \cdot K$$

K- Fischer's information matrix.

$$D(x - \hat{x}) \ge 1/G$$

Cramér–Rao bound

To find the matrix $P_{M\times N}$ *we are use iterational algorithm which start from some initial nondegenerate set of scales:*

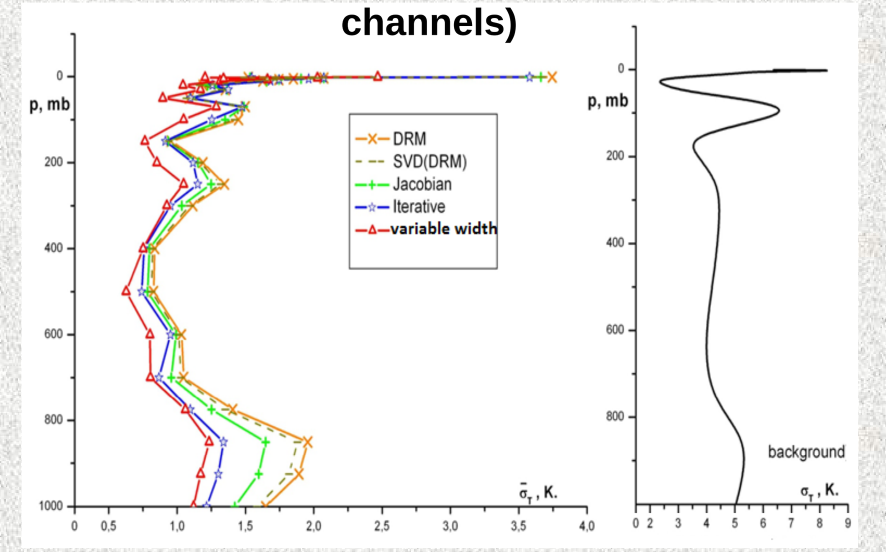
$$\pi_0: \{p_j^0(\nu_i)\}_{j=\overline{1,M}}^{i=\overline{1,N}} \qquad p_j^{(k+1)}(\nu_i) = \begin{cases} \beta_k + (1-\beta_k) p_j^{(k)}(\nu_i), & \text{if } l_j^{(k)}(\nu_i) > 0 \\ (1-\beta_k) p_j^{(k)}(\nu_i), & \text{if } l_j^{(k)}(\nu_i) \le 0 \end{cases}$$

$$0 < \beta_k < 1; \quad l_j^{(k)}(\nu_i) = \sum_{m=1}^M U_m(\nu_i) K^{jm}$$

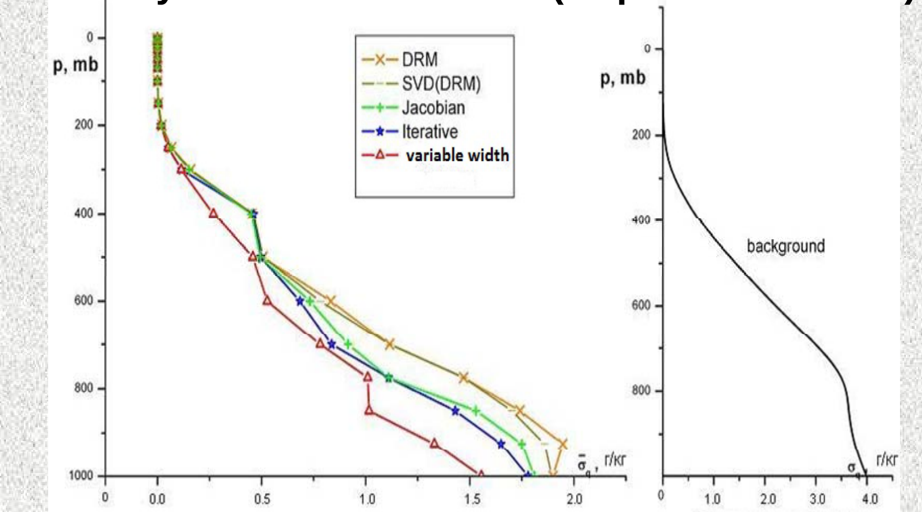
K^{jm} *- cofactor of matrix K.*

Numerical experiments

Temperature retrieval accuracies (23 partial channels)



Humidity retrieval accuracies (23 partial channels)

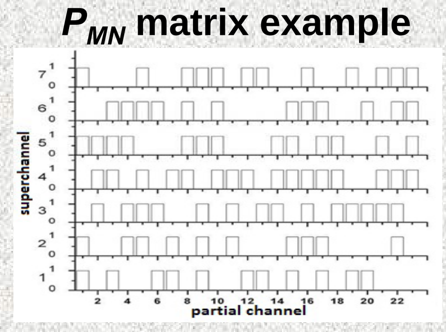


Modeling instrument specifications:

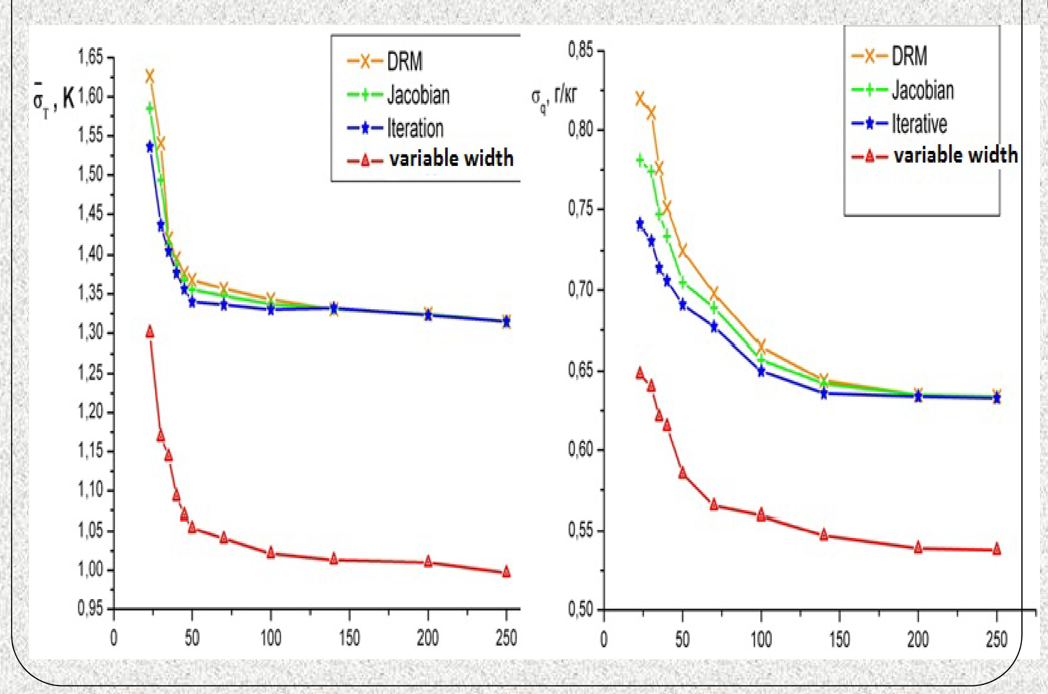
- *Operating range: 650 - 900 см*$^{-1}$*,*
- *-1210- 1650 см*$^{-1}$*, 817-822 см*$^{-1}$
- *Spectral resolution ~ 0.25 см*$^{-1}$
- *Accuracy (NEdT)~ 0.3 K*

(corresponding to IASI (8461 channels)

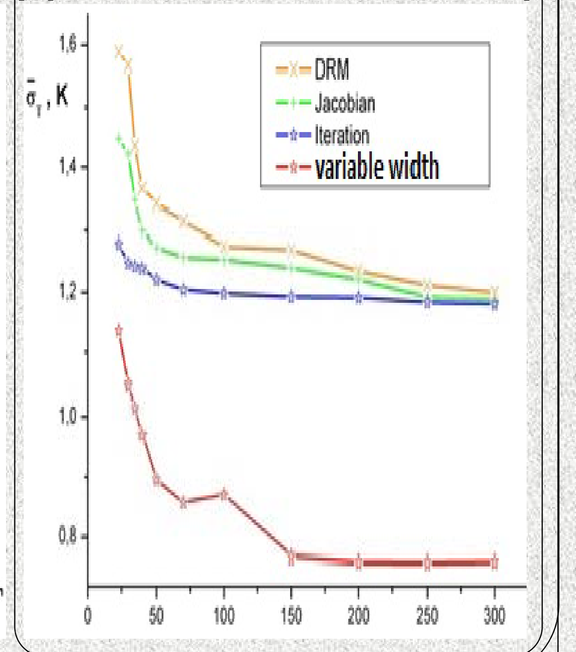
P_{MN} matrix example



Temperature and Humidity retrieval accuracies depending on number of used channels (spectral resolution 0.25cm^{-1} **)**



Temperature retrieval accuracies depending on number of used channels (spectral resolution 0.1 cm^{-1} **)**



Retrieval accuracies (23 channels)		DRM	SVD (DRM)	Jacobian	Iterative	Variable width
LSR	Error of T_0, K	0.41	0.41	0.41	0.41	0.41
LSR	Error of T(z), K	1.78	1.78	1.73	1.64	1.39
LSR	Error of q(z), g/kg	0.83	0.83	0.81	0.79	0.68
Var	Error of T_0, K	0.32	0.32	0.32	0.32	0.32
Var	Error of T(z), K	1.62	1.62	1.59	1.52	1.30
Var	Error of q(z), g/kg	0.82	0.82	0.78	0.74	0.64

Conclusions and perspectives:

- ***The numerical experiments showed the advantage of the method 5 (Channel merging), due to integration of the energy of radiation in various spectral intervals.***
- ***The efficiency of the method increases for better spectral resolution .***
- ***The method suggests an idea of the construction of instrument adopted for the atmospheric remote sensing with spectral intervals merging.***

1. *Uvarov N. Methods of channel selection for contemporary IR-spectrometers // Computational Technologies. Tomsk. 2006, V.11, P.1, PP. 72-77 (in russian).*
2. *Menke W. Geophysical Data Analysis: Discrete Inverse Theory., Academic Press, New York, 1984., 289 p.p.*
3. *Prunet et al. Assimilation of surface data in a onedimensional physical-biogeochemical model of the surface ocean. //Global Biogeochem. Cycles, 10, 1996, 111-138 p.p.*
4. *Rogers C.D. Information content and optimisation of high spectral resolution measurements. //OST&IASRII. SPIE. Volume 2830, 1996, 136-147 p.p.*
5. *Kozlov V.P. About one problem of optimum planning of statistical experiment. //the Theory of probability and its application. 1974. T.19. Page 226-230. (in russian)*