Comparing 4DVar and ensemble-var assimilation methods using a simple model

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Contents

- 4DEnVar - what could we learn?
  - Localization issues in 4DEnVar
  - Can it beat 4D-Var (using a static B)?

Details:

- Work done with PhD student David Fairbairn(Univ. of Surrey) and Andrew Lorenc
- Lorenz 2005 model used (vary resolution of forecast and truth to create model error)
- *"A comparison of 4D-Var with ensemble data assimilation methods"*, D. Fairbairn, S.R. Pring, A.C. Lorenc, I. Roulstone, submitted to QJRMS

Formulation of 4DEnVar equations

In 4DEnVar the background $\underline{\mathbf{x}}_f$ is updated using the equation

$$\underline{\mathbf{x}} = \underline{\mathbf{x}}_f + \delta\underline{\mathbf{x}}$$
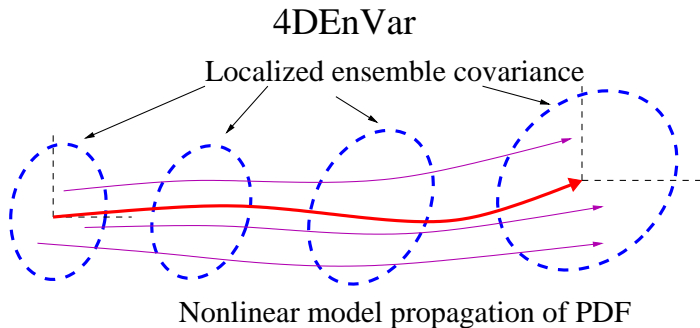
where update $\delta\underline{\mathbf{x}}$ is

$$\delta\underline{\mathbf{x}} = \frac{1}{\sqrt{nEns-1}} \sum_{j=1}^{nEns} \delta\underline{\mathbf{x}}_j^b \circ \underline{\boldsymbol{\alpha}}_j.$$

$\delta\underline{\mathbf{x}}_j^b$ are the ensemble forecast perturbations and $\underline{\boldsymbol{\alpha}}_j \in \mathbb{R}^{nDim}$ a vector of weights. The cost function is

$$J(\underline{\boldsymbol{\alpha}}) = \frac{1}{2} \sum_{j=1}^{nEns} \underline{\boldsymbol{\alpha}}_j^T \underline{C}^{-1} \underline{\boldsymbol{\alpha}}_j + \frac{1}{2}(\underline{y} - \underline{y}^o)^T R^{-1}(\underline{y} - \underline{y}^0)$$

with C a localization function, typically Gaspari-Cohn.

4DEnVar

Localized ensemble covariance

Nonlinear model propagation of PDF

- In NWP a static B matrix is included to provide additional information (sub-sampling issue) - we did not do this in our experiments
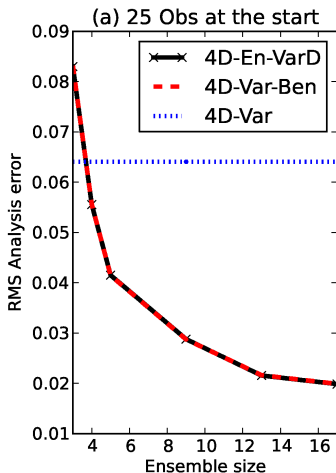- Note that 4DVar-Ben only localizes the $\mathbf{B_{ens}}$ at the start of the window

4DEnVar

Localized ensemble covariance

Nonlinear model propagation of PDF
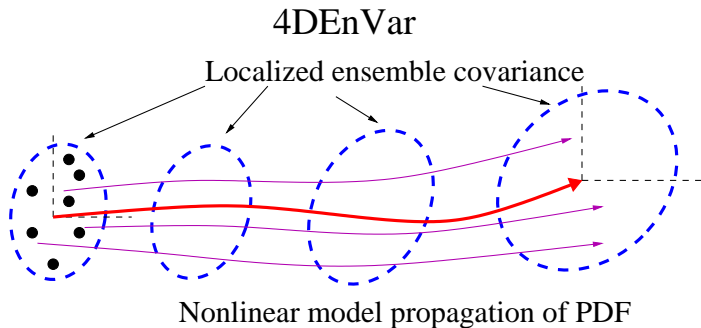
- In NWP a static B matrix is included to provide additional information (sub-sampling issue) - we did not do this in our experiments
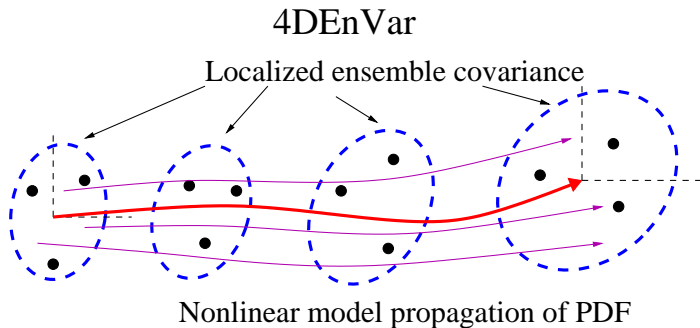- Note that 4DVar-Ben only localizes the $\mathbf{B_{ens}}$ at the start of the window

"Perfect" model result (forecast=truth model)

Demonstrating equivalence of 3DVar-Ben and 3DEnVar

(a) 25 Obs at the start

- All observations at the start of the window
- Plot showing 3DVar-Ben equivalent to 3DEnVar
- Analysis errors calculated against known truth

4DEnVar

Localized ensemble covariance

Nonlinear model propagation of PDF

- In NWP a static B matrix is included to provide additional information (sub-sampling issue) - we did not do this in our experiments
- Note that 4DVar-Ben only localizes the $\mathbf{B_{ens}}$ at the start of the window

"Perfect model" result

Time-distributed obs

(b) 25 Obs even in time
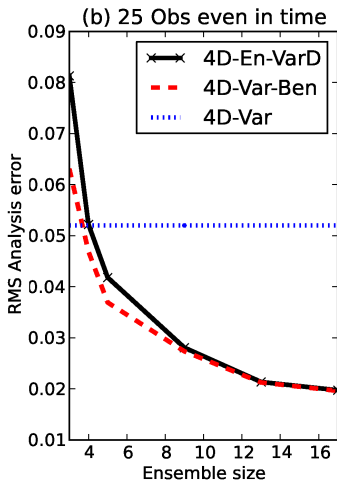
- Place obs at different time-steps to test flow-dependency of B
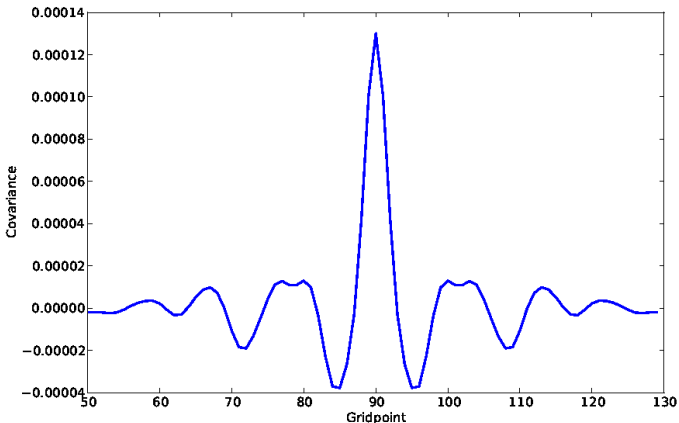- Severe localization during the assimilation window in 4DEnVar is disrupting 4D structure

$$\mathbf{M}_i(C \circ B_{0,ens})\mathbf{M}_i^T \neq C \circ (M_i B_{0,ens} M_i^T)$$

(non-commutative)

Model error correlation

- Truth uses 240 grid points, forecast using 180 grid points. (Cubic interpolation)
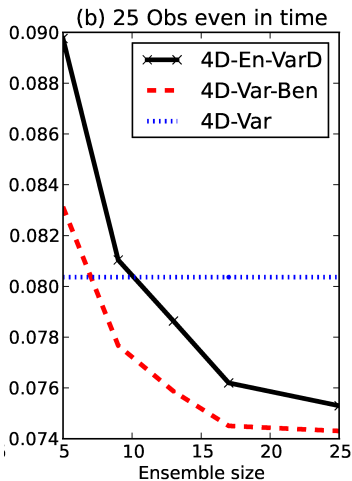- Model error correlation Q:

Model error result

Effect of Model error

(b) 25 Obs even in time

- In this example the ensemble is not accounting for model error - thus incorrect background -> localization

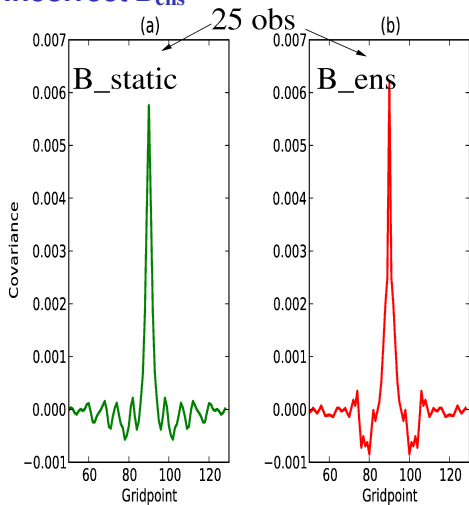- Consistent gap between 4DEnVar(black line) and 4DVar-Ben(red)

$$\mathbf{M}_i(C \circ B_{0,ens})\mathbf{M}_i^T \neq C \circ (M_i B_{0,ens} M_i^T)$$

$$\mathbf{B} = \mathbf{MAM^T} + \mathbf{Q}$$

Model error result
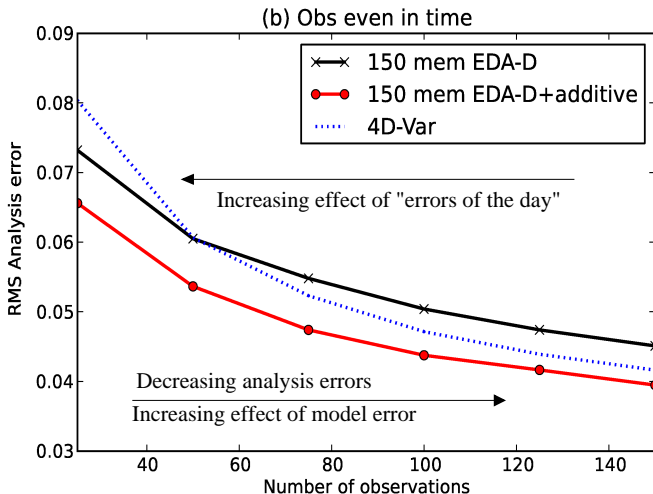
Incorrect B$_{ens}$

25 obs

(a) B_static

(b) B_ens

Model error result

The effect of additive inflation

(b) Obs even in time

Legend:
- 150 mem EDA-D
- 150 mem EDA-D+additive
- 4D-Var

Increasing effect of "errors of the day"

Decreasing analysis errors

Increasing effect of model error

$$\mathbf{B} = \mathbf{M}\mathbf{A}\mathbf{M}^{\mathbf{T}} + \mathbf{Q}$$

Conclusions

- Localization can harm time correlations of background error propagation through the window for 4DEnVar (ensemble-var)
- Capturing model error in the ensemble is important in improving 4DEnVar due to less localization required

Questions and answers

www.metoffice.gov.uk/research/our-scientists/data-assimilation

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