Bayesian Processor of Ensemble Members: combining the Bayesian Processor of Output with Bayesian Model Averaging for reliable ensemble forecasting

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Ensemble Forecasts

...a mean to assess the uncertainty in meteorological forecasts

North American Ensemble Forecasting System

- 21 members from GEM (CMC)
- 21 members from GFS (NCEP)
- Site-located air temperature downscaled from 1-degree grid
- Runs 00Z
- Lead times from +24h to +384h by 24h
- Summer 2008
Observation Dataset and Verification

Canadian Daily Climate Data

- Temperature observation from meteorological station in Jean Lesage Intl. Airport (YQB) in Quebec City over the period 1978-2007
- Temperature observed at 00Z
- [http://www.climat.meteo.gc.ca](http://www.climat.meteo.gc.ca)

Continuous Ranked Probability Score

Reliability
Observation Dataset and Verification

Canadian Daily Climate Data

Continuous Ranked Probability Score

- Comparison of cumulative distribution functions from ensemble forecasts $F$, with observation $y$ through the Heaviside function
- Negatively oriented
  - smaller is better

$$\text{CRPS} = \int_{-\infty}^{\infty} \{F(u) - H(u - y)\}^2 \, du$$

Reliability
Observation Dataset and Verification

Canadian Daily Climate Data

Continuous Ranked Probability Score

Reliability

- Statistical consistency between a priori predicted probabilities and a posteriori observed frequencies of the occurrence
- Reliability measured by the Reliability Component of the CRPS decomposition (Hersbach, 2000)
Why Calibrate Ensemble Predictions?

Verification of raw ensemble forecasts on Summer 2008

- GFS: lowest skill for all lead times
- GEM+GFS: skill closed to GEM
  - No additional benefit in forecasting by using 42 members
- Ensemble predictions less skillful than climatology
- Ensemble predictions are not reliable
Why Calibrate Ensemble Predictions?

Verification of raw ensemble forecasts on Summer 2008

- **Unfortunately** uncertainty underestimated by current ensemble prediction systems (EPS)
- **Unfortunately** ensemble often provided at unsuitable spatial/temporal scales (e.g. for hydrological predictions)
Why Calibrate Ensemble Predictions?

Verification of raw ensemble forecasts on Summer 2008

- Unfortunately uncertainty underestimated by current ensemble prediction systems (EPS)
- Unfortunately ensemble often provided at unsuitable spatial/temporal scales (e.g. for hydrological predictions)

→ Statistical post-processing required to obtain reliable ensemble forecasts at appropriate scales
→ Here: calibration of ensemble predictions from GEM
Notations and Hypotheses

**Notations**
- $t$: valid date
- $S$: ensemble size
- $h$: forecast’s lead time
- $y_t$: predictand (quantity to predict)
- $X_t^{(h)}$: ensemble forecasts

$$X_t^{(h)} = \{X_{t,s}^{(h)} , s = 1, 2, ..., S\}$$

Temporal independance: $h$ is omitted

**Assumptions**
Notations and Hypotheses

Notations

Assumptions

• Ensemble members generated in the same way
  ➔ exchangeability

• Numerical model well suited for predicting an unobserved (latent) variable $\xi_t$ (e.g. gridded temperature)

• Latent variable $\xi_t$ exchangeable with all ensemble members $X_{t,s}$

• $\xi_t$ contains all the information required to predict $y_t$
  ➔ $X_t$ and $y_t$ are conditionally independent
BMA Component (Raftery et al., 2005)

- Law of total probability

\[ p(y_t | X_t) = \int p(y_t | \xi_t, X_t) p(\xi_t | X_t) d\xi_t \]

- As \( X_t \) and \( y_t \) are conditionally independant

\[ p(y_t | X_t) = \int p(y_t | \xi_t) p(\xi_t | X_t) d\xi_t \]

- Since the latent variable is exchangeable with ensemble members

\[ p(\xi_t | X_t) \approx \frac{1}{S} \sum_{s=1}^{S} \delta(\xi_t - X_{t,s}) \]
**BMA Component (Raftery et al., 2005)**

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\[ p(\xi_t|X_t) \approx \frac{1}{S} \sum_{s=1}^{S} \delta(\xi_t - X_{t,s}) \]

**BMA framework:** Non-parametric approximation of the predictive distribution

\[ p(y_t|X_t) \approx \frac{1}{S} \sum_{s=1}^{S} p(y_t|\xi_t = X_{t,s}) \]
BPO Component (Krzysztofowicz, 2004)

Bayes’ rule

\[
p(y_t | \xi_t) \propto p(\xi_t | y_t) p(y_t)
\]

\(p(y_t | \xi_t)\) posterior \hspace{1cm} \(p(\xi_t | y_t)\) likelihood \hspace{1cm} \(p(y_t)\) prior

Prior distribution of the predictand \(=\) climatology

- Temperature: Gaussian distribution
- Estimated from the past 30 years (1978–2007) with a moving window of 5 days around the valid date

\[
p(y_t) = \mathcal{N}(y_t; \mu, \sigma^2)
\]

Likelihood
**BPO Component (Krzysztofowicz, 2004)**

**Likelihood function**

- Assuming linear model with Gaussian residuals between the predictand and the latent variable

\[
\xi_t = \alpha + \beta y_t + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(0, \sigma_{\epsilon|y}^2)
\]

- But \(\xi_t\) not observable

\[
\bar{X}_t = \alpha + \beta y_t + \eta_t \quad \eta_t \sim \mathcal{N}(0, \sigma_{\eta|y}^2)
\]

- Bayesian specification of the linear regression with informative conjugate priors

- Less information in \(\bar{X}_t\) than in \(\xi_t\) :

\[
\sigma_{\epsilon|y}^2 \in \left[0; \sigma_{\bar{X}|y}^2\right]
\]

\[\Rightarrow\] Optimal variance \(\sigma_{\epsilon|y}^2\) estimated by minimizing the CRPS
BPO Component (Krzysztofowicz, 2004)

**Bayes’ rule**

\[
p(y_t | \xi_t) \propto p(\xi_t | y_t) p(y_t)
\]

\text{posterior} \quad \text{likelihood} \quad \text{prior}

**Prior**

\[
p(y_t) = \mathcal{N}(y_t; \mu, \sigma^2)
\]

**Likelihood**

\[
p(\xi_t | y_t) = \mathcal{N}\left(\xi_t; \alpha + \beta y_t, \sigma_{\epsilon|y}^2\right)
\]
BPO Component (Krzysztofowicz, 2004)

Bayes’ rule

\[ p(y_t | \xi_t) \propto p(\xi_t | y_t) p(y_t) \]

- **Posterior**
- **Likelihood**
- **Prior**

Prior

\[ p(y_t) = \mathcal{N}(y_t; \mu, \sigma^2) \]

Likelihood

\[ p(\xi_t | y_t) = \mathcal{N}(\xi_t; \alpha + \beta y_t, \sigma_{\epsilon | y}^2) \]

**BPO framework:**

\[ p(y_t | \xi_t) = \mathcal{N}(y_t; \mu_{y | \xi}, \sigma_{y | \xi}^2) \]

\[
\mu_{y | \xi} = \frac{\sigma^2 \beta^2 (\xi_t - \frac{\alpha}{\beta}) + \sigma_{\epsilon | y}^2 \mu}{\sigma^2 \beta^2 + \sigma_{\epsilon | y}^2} \\
\sigma_{y | \xi}^2 = \frac{\sigma^2 \sigma_{\epsilon | y}^2}{\sigma^2 \beta^2 + \sigma_{\epsilon | y}^2}
\]
Bayesian Processor of Ensemble Members

### BMA

\[
p(y_t | X_t) \approx \frac{1}{S} \sum_{s=1}^{S} p(y_t | \xi_t = X_{t,s})
\]

### BPO

\[
p(y_t | \xi_t) = \mathcal{N}(y_t; \mu_{y|\xi}, \sigma_{y|\xi}^2)
\]

### BPEM

\[
p(y_t | X_t) \approx \frac{1}{S} \sum_{s=1}^{S} \mathcal{N}(y_t; \mu_{y|X_{t,s}}, \sigma_{y|X_{t,s}}^2)
\]

\[
\mu_{y|X_{t,s}} = \frac{\sigma^2 \beta^2 \left( \frac{X_{t,s} - \alpha}{\beta} \right) + \sigma_{\epsilon|y}^2 \mu}{\sigma^2 \beta^2 + \sigma_{\epsilon|y}^2}
\]

\[
\sigma_{y|X_{t,s}}^2 = \frac{\sigma^2 \sigma_{\epsilon|y}^2}{\sigma^2 \beta^2 + \sigma_{\epsilon|y}^2}
\]
Bayesian Processor of Ensemble Members

\[ p(y_t|X_t) \approx \frac{1}{S} \sum_{s=1}^{S} \mathcal{N}(y_t; \mu_{y|X_t,s}, \sigma^2_{y|X_t,s}) \]

\[ \mu_{y|X_t,s} = \frac{\sigma^2 \beta^2 \left( \frac{X_{t,s} - \alpha}{\beta} \right) + \sigma^2_{\epsilon|y} \mu}{\sigma^2 \beta^2 + \sigma^2_{\epsilon|y}} \]

\[ \sigma^2_{y|X_t,s} = \frac{\sigma^2 \sigma^2_{\epsilon|y}}{\sigma^2 \beta^2 + \sigma^2_{\epsilon|y}} \]

- \( \mu_{y|X_t,s} \): weighted average of bias-corrected member \( \frac{X_{t,s} - \alpha}{\beta} \) and of the prior mean from climatology \( \mu \)
- \( \sigma^2_{y|X_t,s} \): weighted mixture of residuals variance of the linear model \( \sigma^2_{\epsilon|y} \) and of the climatological variance \( \sigma^2 \)
Bayesian Processor of Ensemble Members

- The predictive distributions’s shape depends on the empirical distribution of the ensemble members

  ➡️ the predictive distribution is not necessary Gaussian

![Graphs showing predictive distributions for different dates and conditions](image)

- GEM / calibrated members
- Observation
- Predictive dist.
- Constituent dist.
Identification of the Optimum Training Length

Summer 2008

• training period: 10-days to 50-days joint samples
• No significant gain in forecasts’s skill beyond 15 days

<y> log scale

• Calibrated forecasts less skillful than climatology-based forecasts for longer-range lead times
• Optimal training length: 15 days
• Short training length: limit effects of seasonality and frequent changes to operational forecasting systems
Verification of Calibrated Ensemble Forecasts

Summer 2008 (Québec City only)

- Optimal training length for each calibration method
- Forecasts’ skill improved by calibration (up to +192h) with similar pattern for each calibration method
- Significant improvement of forecasts’ reliability

(y) log scale
Verification of Calibrated Ensemble Forecasts

Summer 2008 (8 sites in Québec)

+24h

+96h

CRPS

CRPS Reliability

BC CH GP MTL QC SH SI VO

CRPS

CRPS Reliability

BC CH GP MTL QC SH SI VO

GEM Clim. BPEM BPO ensBMA
Conclusions

Bayesian Processor of Ensemble Members... 

...a new approach to calibrated ensemble forecasts

- Based on BMA and BPO frameworks
- Capable to generate reliable forecasts
- Outperforms slightly both the BMA and BPO approaches as well as a climatology
- Short optimal training length: avoid negative impacts of seasonality and of frequent changes to operational forecasting systems
- Successfully applied to 7 other stations across the Quebec
Thanks for your attention