Pseudo-orbit gradient descent ensemble data assimilation

Hailiang Du, Leonard A. Smith and Emma Suckling

Centre for the Analysis of Time Series, London School of Economics

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Pseudo-orbit gradient descent ensemble assimilation

Outline

- **The pseudo-orbit DA (PDA) methodology**
  - **Differences with other methods**
- **Low-dimensional model examples**
  - **Nowcasting with ensembles in the PMS and IMS**
  - **Ikeda (2D) and Lorenz96 (18D) systems**
  - **Comparison with EnKF, 4DVAR**
- **Extensions and further examples**
- **Open questions**
- **TEMIP??** ☺

Pseudo-orbit gradient descent ensemble assimilation

Motivation

- **With data assimilation we aim to gain an estimate the current state**
    - **Partial, noisy observations are incorporated into imperfect models**
    - **Trajectories are generated, ideally consistent with measurements and model dynamics**
- **Some approaches place more weight on the observations**
    - **Relying less on the model dynamics**
- **Some approaches place more weight on the model dynamics**
    - **Making assumptions about the model error**
- **PDA aims at a better balance between observations and dynamics**
    - **Placing more weight on model dynamics with minimal assumptions of form of model error**

Pseudo-orbit gradient descent ensemble assimilation

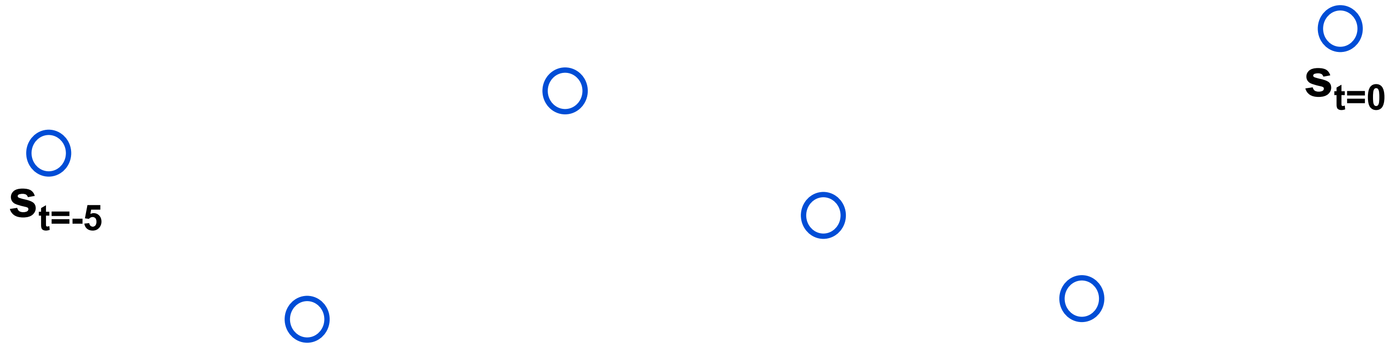
Terminology

- **Let $x_t \in \mathbb{R}^m$ be a model state vector**
- X **is a point in an** $m \times n$ **sequence space corresponding to a set of** x_t
- $F(x_t) = x_{t+1}$
  - **Maps** x_t **at** t **into** x_{t+1}
  - **Defines a trajectory**
- U **defines a pseudo-orbit in sequence space with components** $u_t \in \mathbb{R}^m$
- **Let** s_t **be an observation of state** x_t **with some additive noise** ε_t
- $t = 1,2,\ldots n$

Pseudo-orbit gradient descent DA method

- **Start with a pseudo-orbit defined by the noisy observations**

$$^{0}U = S$$



Kevin Judd, Leonard Smith and Antje Weisheimer, Physica D 190 (2004)

Pseudo-orbit gradient descent DA method

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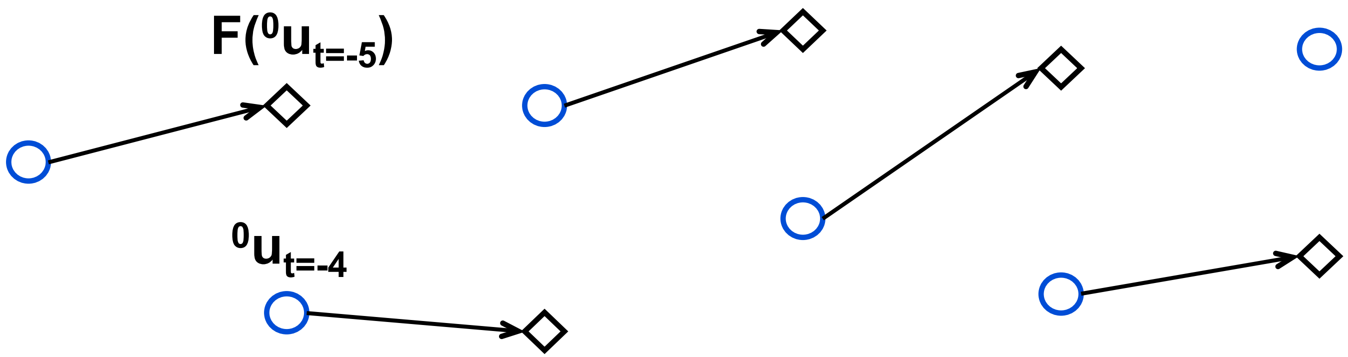
- **Generate 1-step ahead trajectories**



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Pseudo-orbit gradient descent DA method

- **Start with a pseudo-orbit defined by the noisy observations**

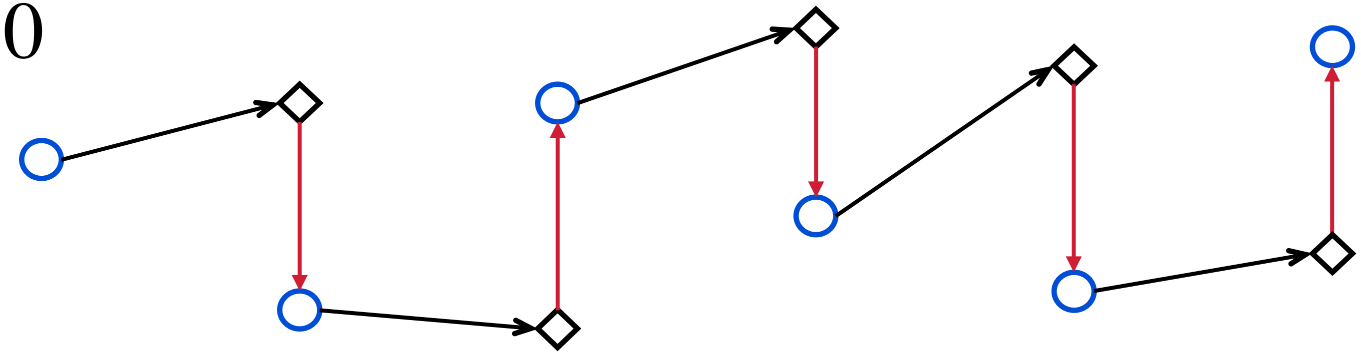
$$^{0}U = S$$

- **Generate 1-step ahead trajectories**

- **Define mismatch function and minimise**

$$C_{PDA}(^{\alpha}U) = \sum_{t=1}^{n} \left| F(^{\alpha}u_t) - {}^{\alpha}u_{t+1} \right|^2$$

$$\alpha = 0$$



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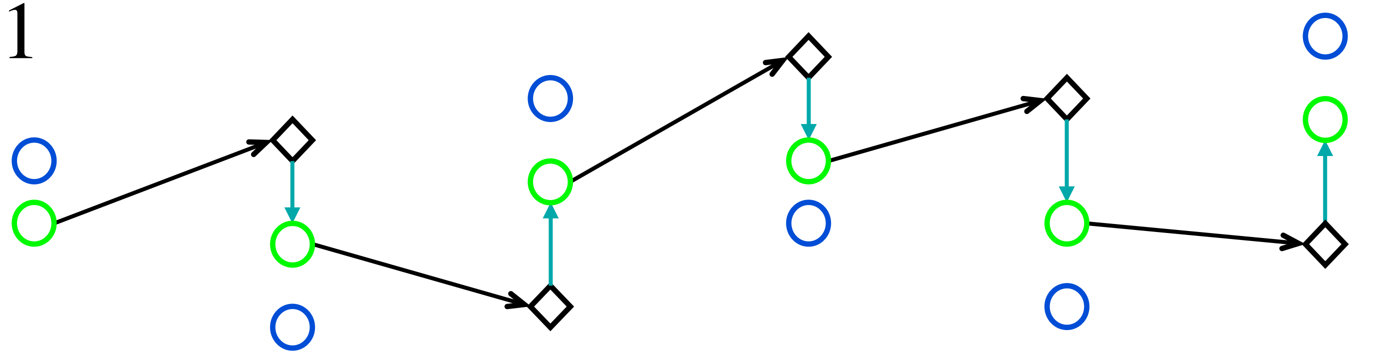
$$C_{PDA}(^{\alpha}U) = \sum_{t=1}^{n} \left| F(^{\alpha}u_t) - {}^{\alpha}u_{t+1} \right|^2$$

$$\alpha = 1$$



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Pseudo-orbit gradient descent DA method

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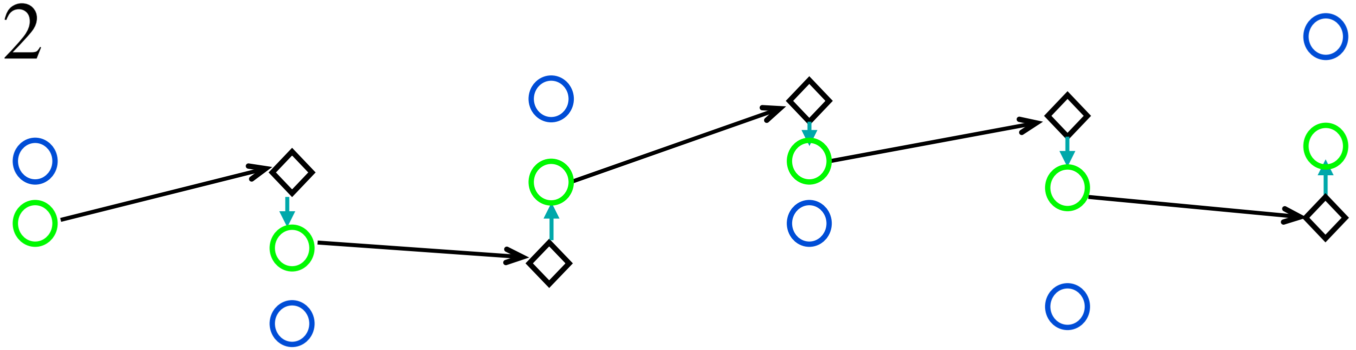
$$C_{PDA}(^{\alpha}U) = \sum_{t=1}^{n} \left| F(^{\alpha}u_t) - {}^{\alpha}u_{t+1} \right|^2$$

$$\alpha = 2$$



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Pseudo-orbit gradient descent DA method

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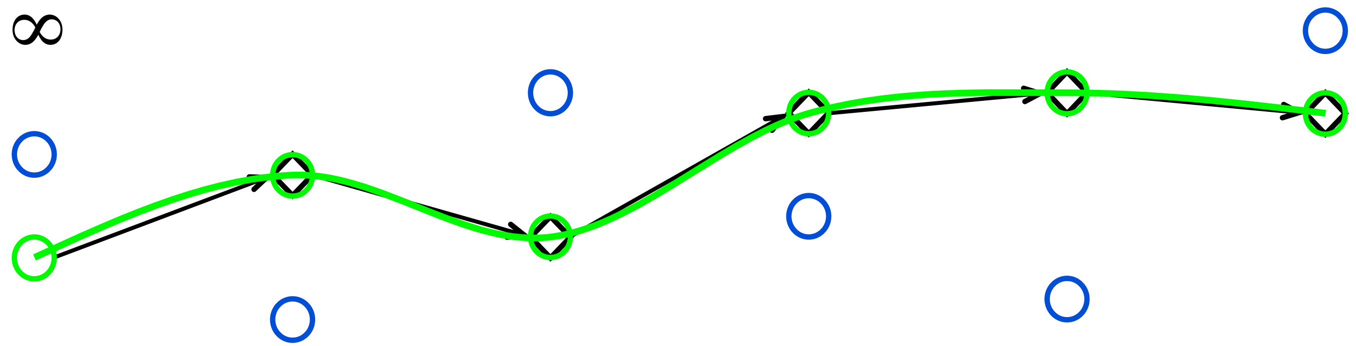
$\alpha \to \infty$



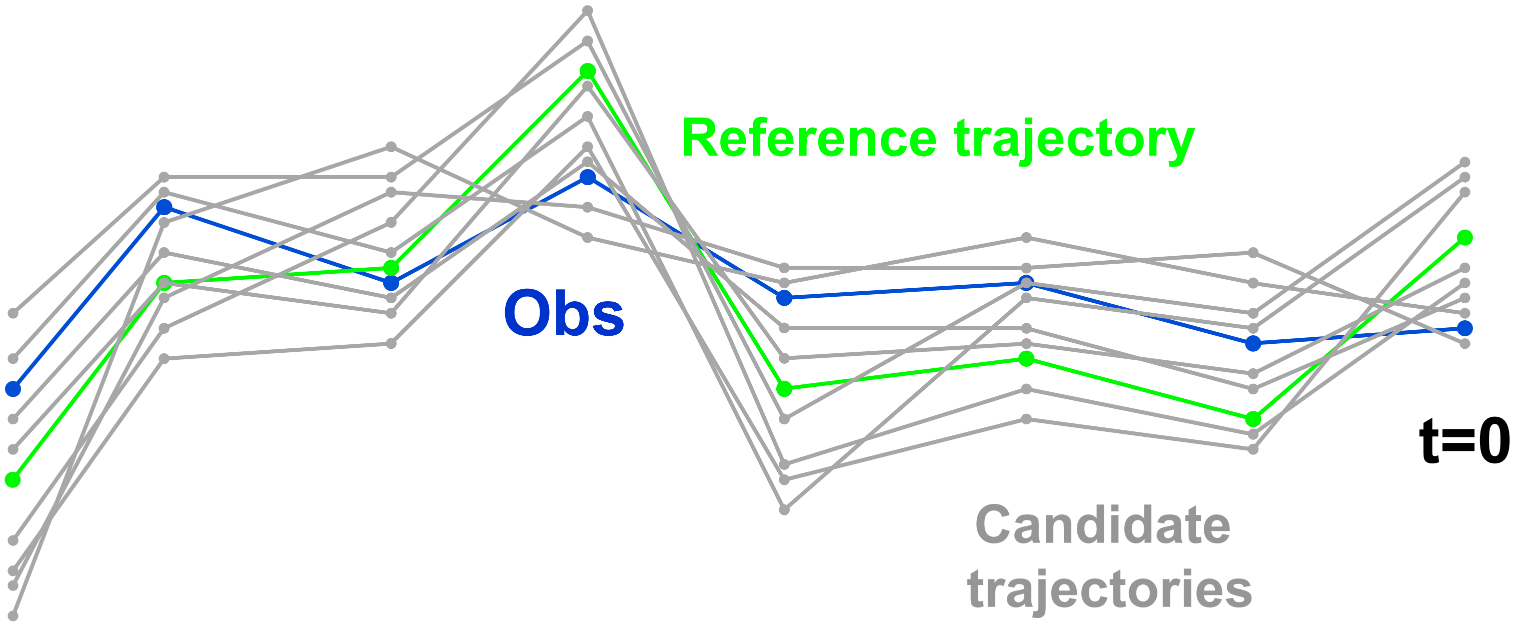
- **The pseudo-orbit *U* converges to a trajectory as** $\alpha \to \infty$

Pseudo-orbit gradient descent DA - Advantages

- **PDA is a smoother rather than a filter**
  - **Limits the impact of single ‘bad’ observations**
- **There are no local minima**
  - **Each global minimum is a trajectory of the model**
- **More reliance on model dynamics**
  - **Advantageous for long assimilation windows**
  - **Does not attempt to stick too closely to the observations**
  - **Observations are used to define initial model pseudo-orbit**
- **Doesn’t assume structure of model error is known**
- **Fully nonlinear**
  - **No assumptions/requirements for linear dynamics or Gaussian distributions**

Generating ensembles

- **Like 4DVAR we must still generate an ensemble**
- **There are several approaches to generating candidate trajectories**
  - **Start from perturbations on the observations and do PDA**
  - **Sample the local space**



Generating ensembles

- **Like 4DVAR we must still generate an ensemble**
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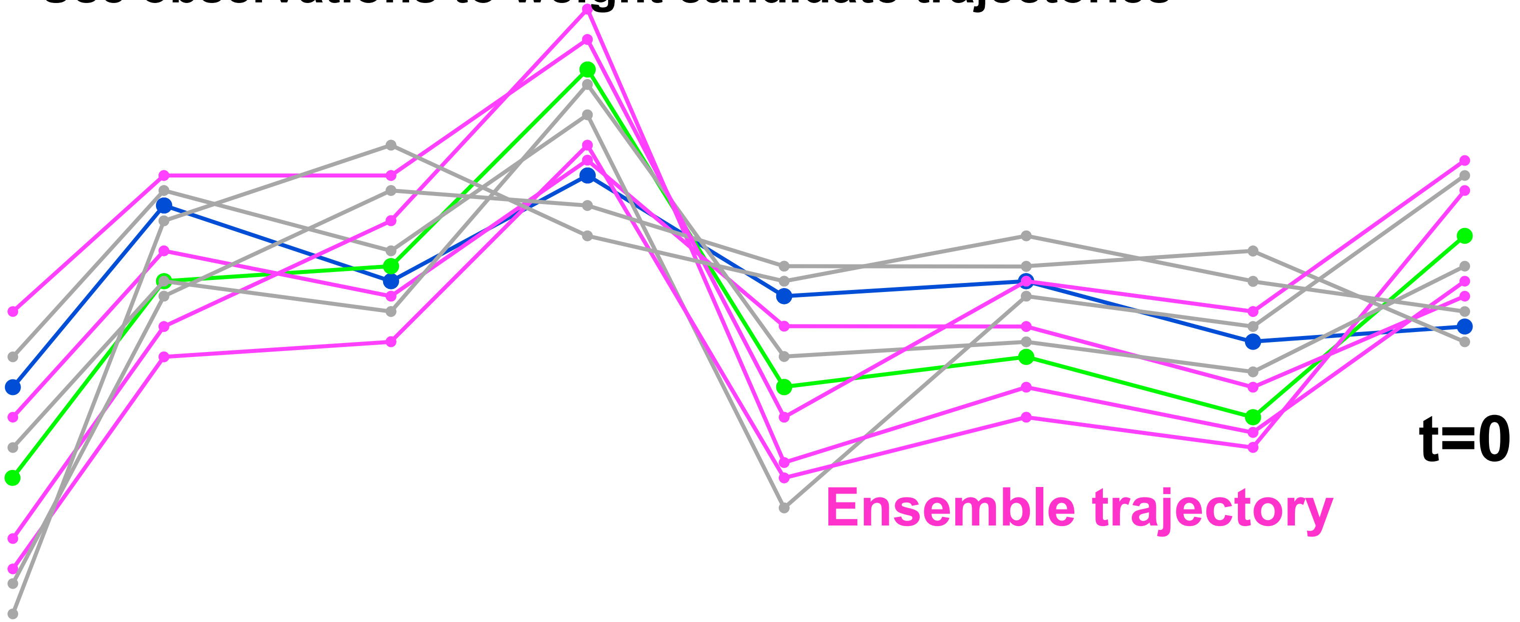
  - **Start from perturbations on the observations and do PDA**
  - **Sample the local space**
- **Use observations to weight candidate trajectories**



Generating ensembles

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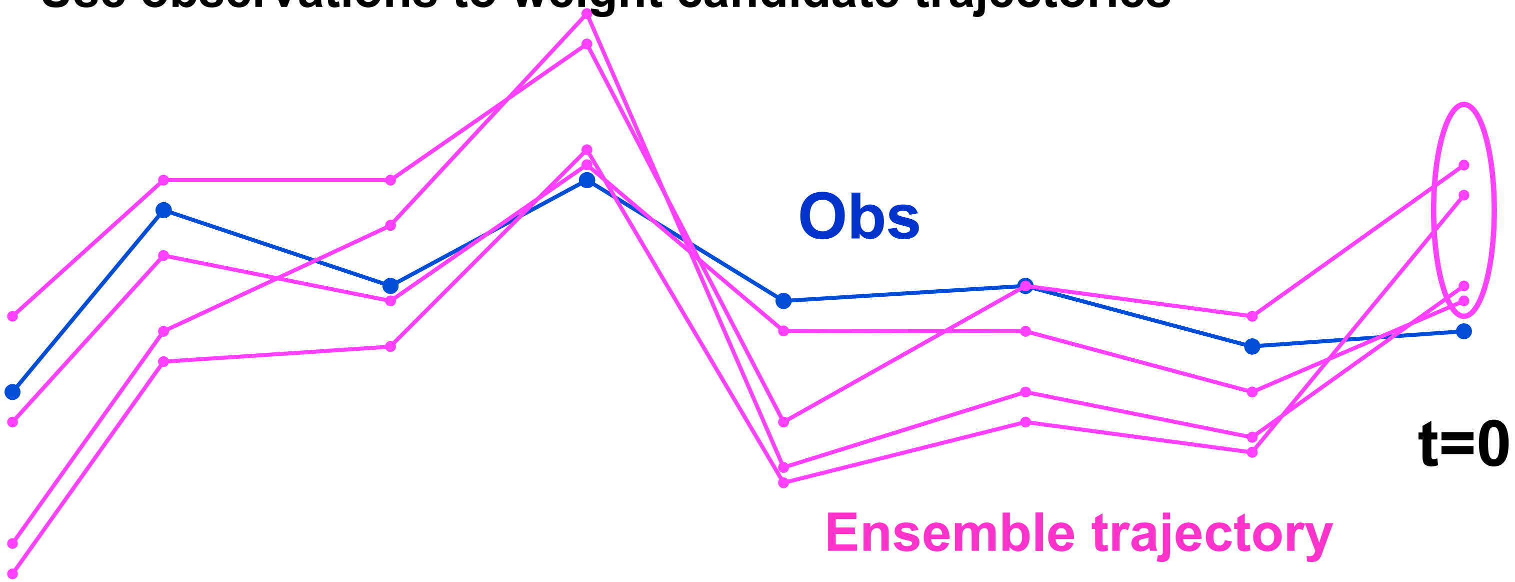


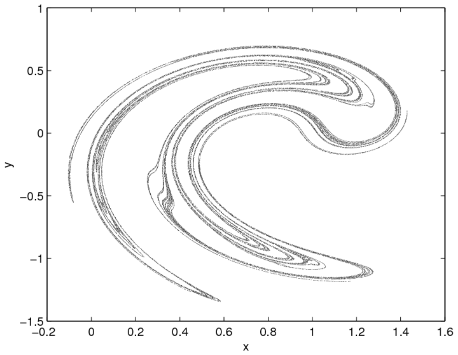
Pseudo-orbit gradient descent DA in PMS

- **Compare nowcast ensemble members from PDA and EnKF**
  - **Ikeda system (2D) → Noise model N(0,0.4)**
  - **Lorenz96 system (18D) → Noise model N(0,0.05)**
  - **Generate 512 ensemble members**

- **Evaluate Ignorance score:**

$$S(p(y), Y) = -log(p(Y))$$

- **Compare ensemble members from PDA and 4DVAR**
  - **Ikeda system over different window lengths**
  - **Ensemble members generated in identical way**



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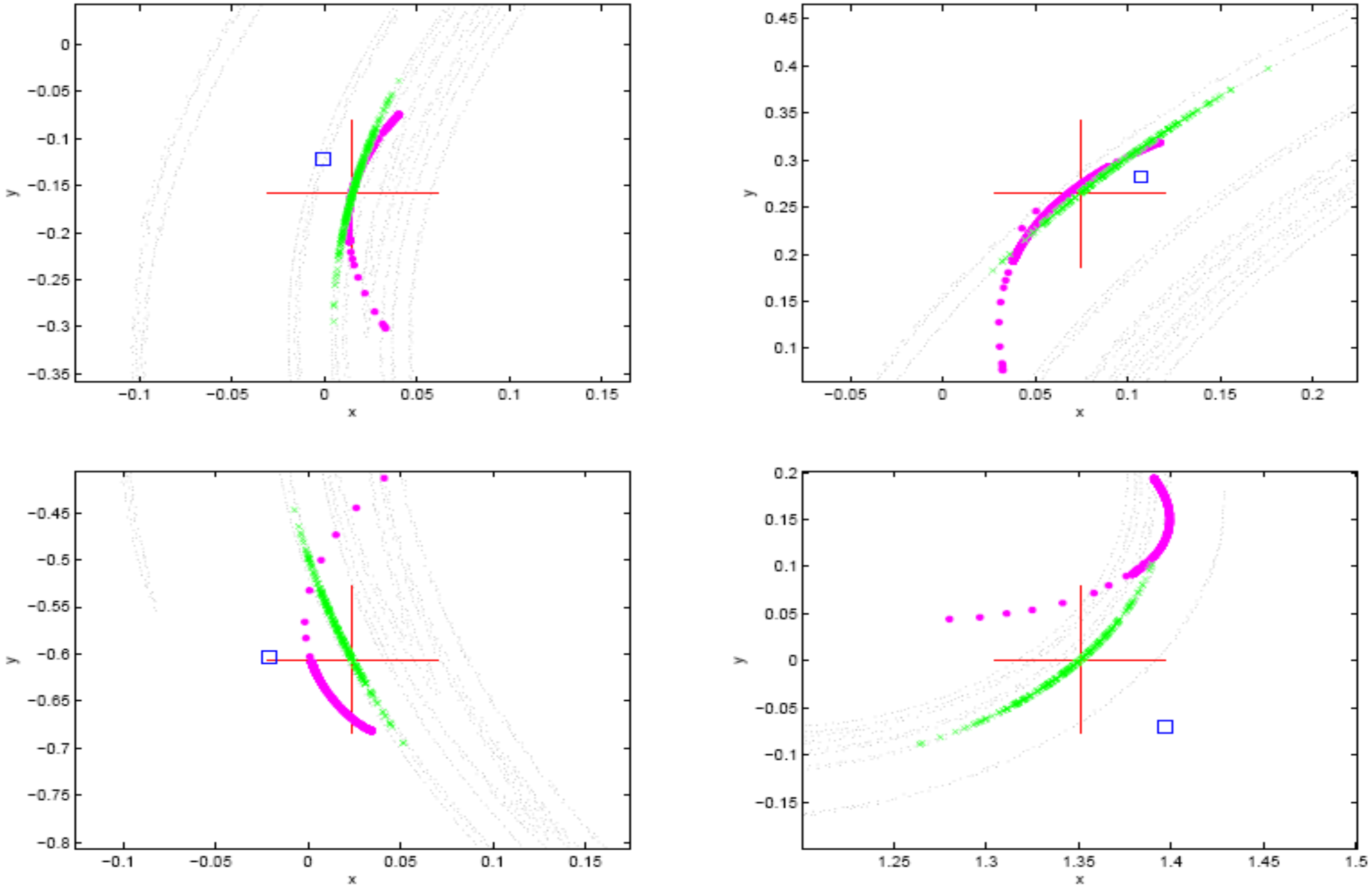
Pseudo-orbit gradient descent DA in PMS



Nowcast ensemble (512 members) of the Ikeda map
Pink: EnKF ensemble Green: PDA ensemble Blue: observation

Pseudo-orbit gradient descent DA in PMS

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  - **Lorenz96 system → Noise model N(0,0.05)**
  - **Generate 512 ensemble members**
- **Evaluate Ignorance score:**

$$S(p(y), Y) = -log(p(Y))$$

  - **Lower and upper are 90th percent bootstrap resampling bounds**
  - **Lower scores indicate more skill**
- **On average PDA outperforms EnKF by ~1.5 bit**

Systems	Ignorance		Lower		Upper		Kernel width	
	EnKF	PDA	EnKF	PDA	EnKF	PDA	EnKF	PDA
Ikeda	-3.21	-4.67	-3.28	-4.75	-3.13	-4.60	0.0290	0.0011
Lorenz96	-3.72	-4.44	-3.78	-4.49	-3.66	-4.38	0.28	0.07

Pseudo-orbit gradient descent DA in PMS

- **Compare nowcast ensemble members from PDA and 4DVAR**
  - **Ikeda system over different window lengths**
  - **Ensemble members generated in identical way**
- **PDA ensembles closer to truth on average than 4DVAR over long window**

Window length	a) Distance from observations					
	Average		Lower		Upper	
	4DVAR	PDA	4DVAR	PDA	4DVAR	PDA
4 steps	1.58	1.66	1.51	1.59	1.63	1.73
6 steps	11.06	1.77	8.17	1.71	14.28	1.83
8 steps	51.84	1.85	46.16	1.80	58.54	1.90
Window length	**b) Distance from truth**					
	Average		Lower		Upper	
	4DVAR	PDA	4DVAR	PDA	4DVAR	PDA
4 steps	0.52	0.61	0.48	0.55	0.55	0.67
6 steps	9.51	0.39	6.70	0.36	12.59	0.42
8 steps	50.04	0.28	43.59	0.25	55.77	0.31

The imperfect model scenario

Examples

- **Compare nowcast ensemble members from PDA and EnKF**
  - **Ikeda model-system and Lorenz96 model-system pairs**
  - **Model dynamics and and observations generated from different systems**
- **Find pseudo-orbit of imperfect model, *f*, consistent with observations**
  - **A stopping criteria is needed to find a consistent reference trajectory**

PDA stopping criteria for IMS

- **Define implied noise:**

$$\delta_i = \mathbf{s}_i - \mathbf{u}_i$$

- **Define imperfection error:**

$$\omega_i = \mathbf{u}_i - f(\mathbf{u}_{i-1})$$

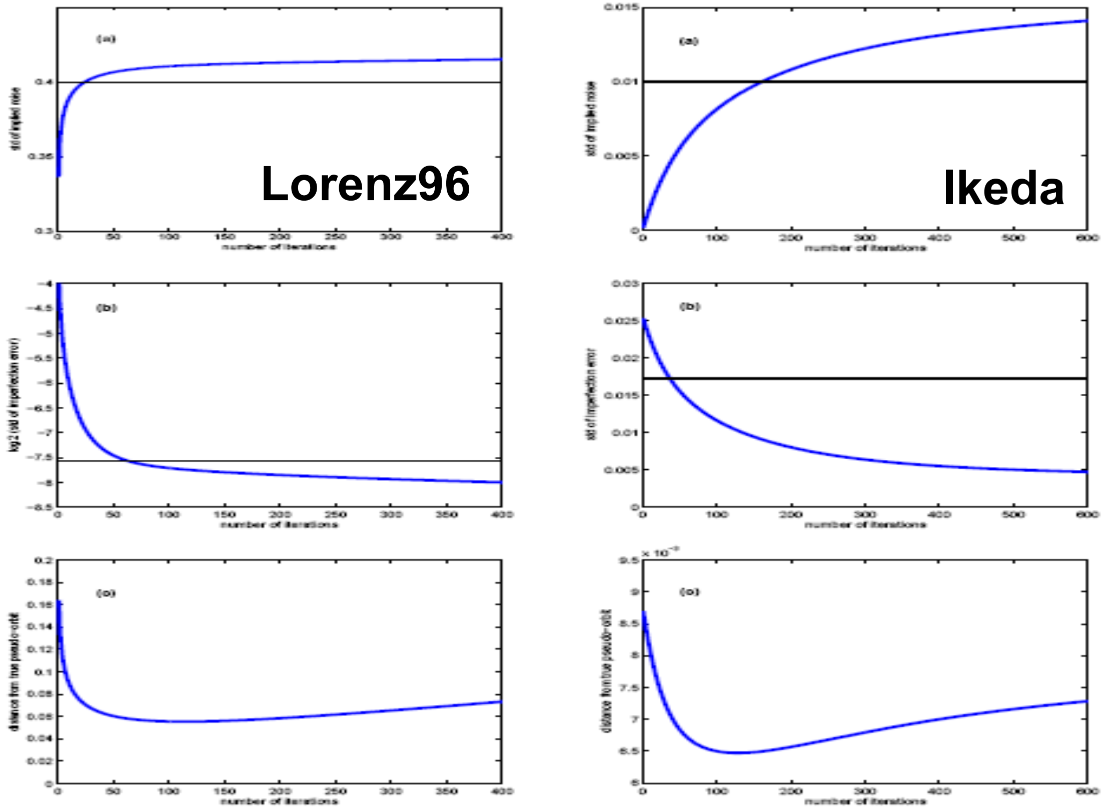
Pseudo-orbit gradient descent DA in IMS

PDA stopping criteria

Implied noise

Imperfection error

Distance from the "truth"



Pseudo-orbit statistics as a function of gradient descent iterations

Pseudo-orbit gradient descent DA in IMS

Systems	Ignorance		Lower		Upper	
	EnKF	PDA	EnKF	PDA	EnKF	PDA
Ikeda	-2.67	-3.62	-2.77	-3.70	-2.52	-3.55
Lorenz96	-3.52	-4.13	-3.60	-4.18	-3.39	-4.08

Ikeda system-model pair and Lorenz96 system-model pair, the noise model is N(0, 0.5) and N(0, 0.05) respectively. Lower and Upper are the 90% bootstrap resampling bounds of Ignorance score

Pseudo-orbit gradient descent ensemble assimilation

Summary and open questions

- **PDA is a fully nonlinear approach to DA**
- **It is demonstrated to outperform EnKF and 4DVAR in low dimensional examples**
- **Further examples include:**
  - **Lagrangian DA in point-vortex system with partial observations**
  - **Operational NOGAPS model**
  - **Extensions to method including gradient-free descent (limited derivative information)**
- **PDA is designed for imperfect model scenario**
  - **It provides informative estimates for model imperfection**
  - **Requires a stopping criteria – How best to do this?**
- **Further comparisons and examples → TEMIP?**

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Thank You!

Contact Me

Emma Suckling: Centre for the Analysis of Time Series
London School of Economics

e.suckling@lse.ac.uk

Hailiang Du: **h.l.du@lse.ac.uk**

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