# Ensemble Copula Coupling:

# Towards Physically Consistent, Calibrated Probabilistic Forecasts of Spatio-Temporal Weather Trajectories

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## Statistical postprocessing of numerical weather prediction (NWP) ensembles

NWP **ensembles** are subject to **biases** and typically they show a **lack of calibration**

thus, some form of **statistical postprocessing** is required in order to properly quantify uncertainty, and generate **calibrated** and **sharp** predictive distributions

major approaches to the **statistical postprocessing** of **NWP ensembles** include

- **Bayesian model averaging (BMA)**, which fits a mixture density as predictive PDF, where each ensemble member is associated with a kernel function, using a weight that reflects the member's skill (Raftery et al. 2005)

- **ensemble model output statistics (EMOS)** or **nonhomogeneous Gaussian regression (NGR)**, which fits a single, parametric predictive PDF using summary statistics from the ensemble (Gneiting et al. 2005)

## BMA and EMOS/NGR for temperature

consider an **ensemble forecast**, $x_1, \ldots, x_M$, for **surface temperature**, $y$, at a given location and look-ahead time

- **BMA** employs Gaussian kernels with a linearly bias-corrected mean, i.e., the BMA predictive PDF is the **Gaussian mixture**

$$f(y \,|\, x_1, \ldots, x_M) = \sum_{m=1}^{M} w_m \mathcal{N}(a_{0m} + a_{1m} x_m, \, \sigma^2)$$

  with BMA weights $w_1, \ldots, w_M$, bias parameters $a_{0m}, \ldots, a_{0M}$ and $a_{1m}, \ldots, a_{1M}$ and a common spread parameter $\sigma^2$

- **EMOS/NGR** employs a **single Gaussian** predictive PDF, in that

$$f(y \,|\, x_1, \ldots, x_M) = \mathcal{N}(c_0 + c_1 x_1 + \cdots + c_M x_M, \, d_0 + d_1 s^2)$$

  with location parameters $c_0$ and $c_1, \ldots, c_M$, and spread parameters $d_0$ and $d_1$, where $s^2$ is the ensemble variance

in our experience, the two approaches yield nearly the same predictive performance, with **BMA** being the more **flexible** and **EMOS/NGR** being the more **parsimonious** method

# BMA and EMOS/NGR for univariate weather quantities

## Bayesian model averaging (BMA)

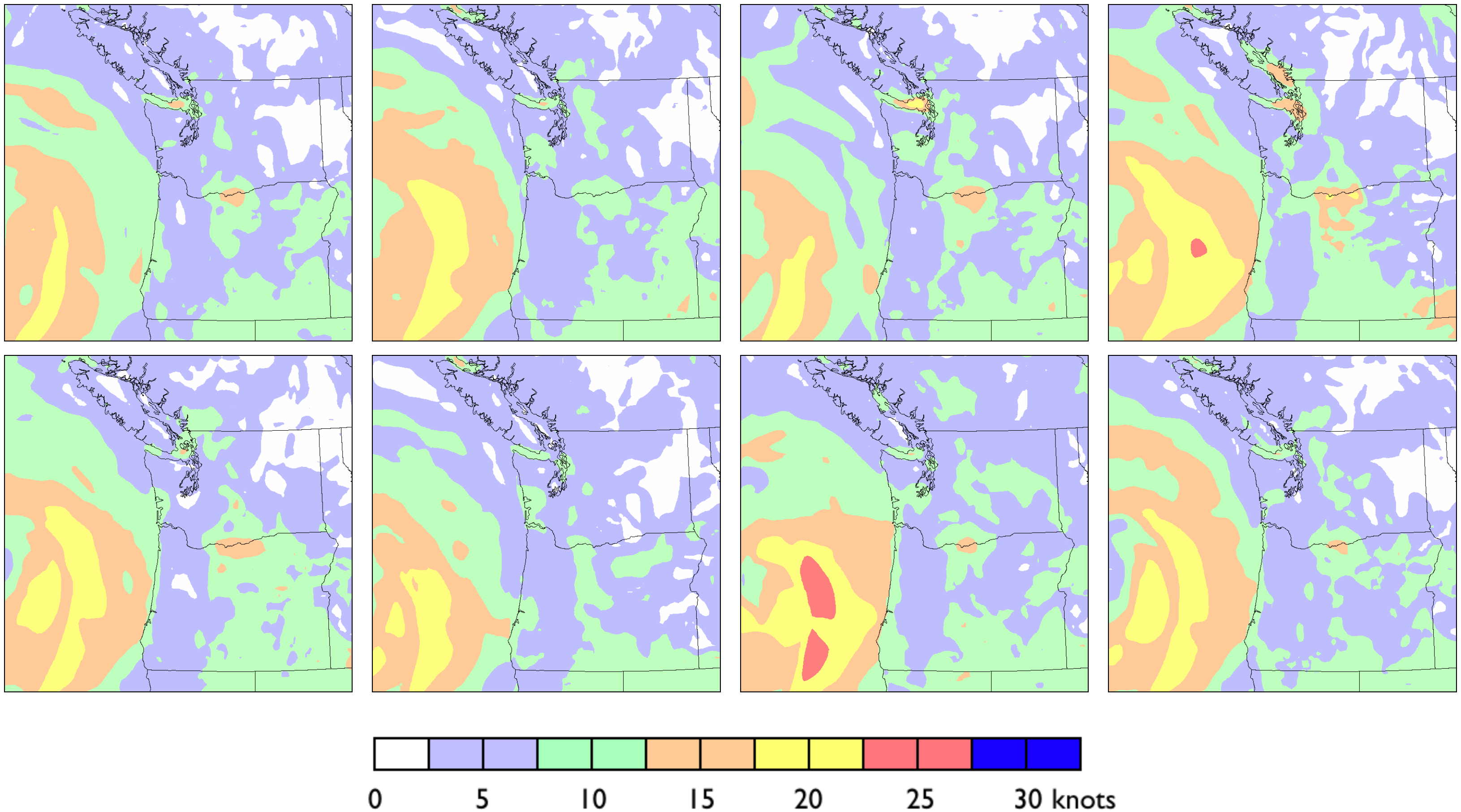
| Variable | Range | Kernel | Mean | Variance |
|---|---|---|---|---|
| Temperature | $y \in \mathbb{R}$ | Normal | $a_{0m} + a_{1m} x_m$ | $\sigma^2$ |
| Pressure | $y \in \mathbb{R}$ | Normal | $a_{0m} + a_{1m} x_m$ | $\sigma^2$ |
| Precipitation accumulation | $y^{1/3} \in \mathbb{R}^+$ | Gamma | $a_{0m} + a_{1m} x_m$ | $b_0 + b_1 x_m$ |
| Wind speed | $y \in \mathbb{R}^+$ | Gamma | $a_{0m} + a_{1m} x_m$ | $b_0 + b_1 x_m$ |
| Visibility | $y \in [0, 1]$ | Beta | $a_{0m} + a_{1m} x_m^{1/2}$ | $b_0 + b_1 x_m^{1/2}$ |

## Ensemble model output statistics (EMOS/NGR)

| Variable | Range | Density | Location | Scale |
|---|---|---|---|---|
| Temperature | $y \in \mathbb{R}$ | Normal | $c_0 + c_1 x_1 + \cdots + c_m x_m$ | $d_0 + d_1 s^2$ |
| Pressure | $y \in \mathbb{R}$ | Normal | $c_0 + c_1 x_1 + \cdots + c_m x_m$ | $d_0 + d_1 s^2$ |
| Wind speed | $y \in \mathbb{R}^+$ | Truncated normal | $c_0 + c_1 x_1 + \cdots + c_m x_m$ | $d_0 + d_1 s^2$ |

## Example: EMOS/NGR for wind speed

48-hour **EMOS/NGR postprocessed** predictive **PDFs** of **wind speed** based on the eight-member University of Washington Meso-scale Ensemble (**UWME**; Eckel and Mass 2005)




48-hour UWME forecast of maximum wind speed valid August 7, 2003

## Example: EMOS/NGR for wind speed

48-hour **EMOS/NGR postprocessed** predictive **PDFs** of **wind speed** over the US **Pacific Northwest** (Thorarinsdottir and Gneiting 2010)

the **EMOS/NGR** predictive PDF is **truncated normal**,

$$f(y \,|\, x_1, \ldots, x_8) \;=\; \mathcal{N}_{[0,\infty)}(c_0 + c_1 x_1 + \cdots + c_8 x_8,\, d_0 + d_1 s^2),$$

with location parameters $c_0$ and $c_1, \ldots, c_8$, and spread parameters $d_0$ and $d_1$, where $s^2$ is the ensemble variance

minimum CRPS estimation of the EMOS/NGR parameters using a **rolling training period**

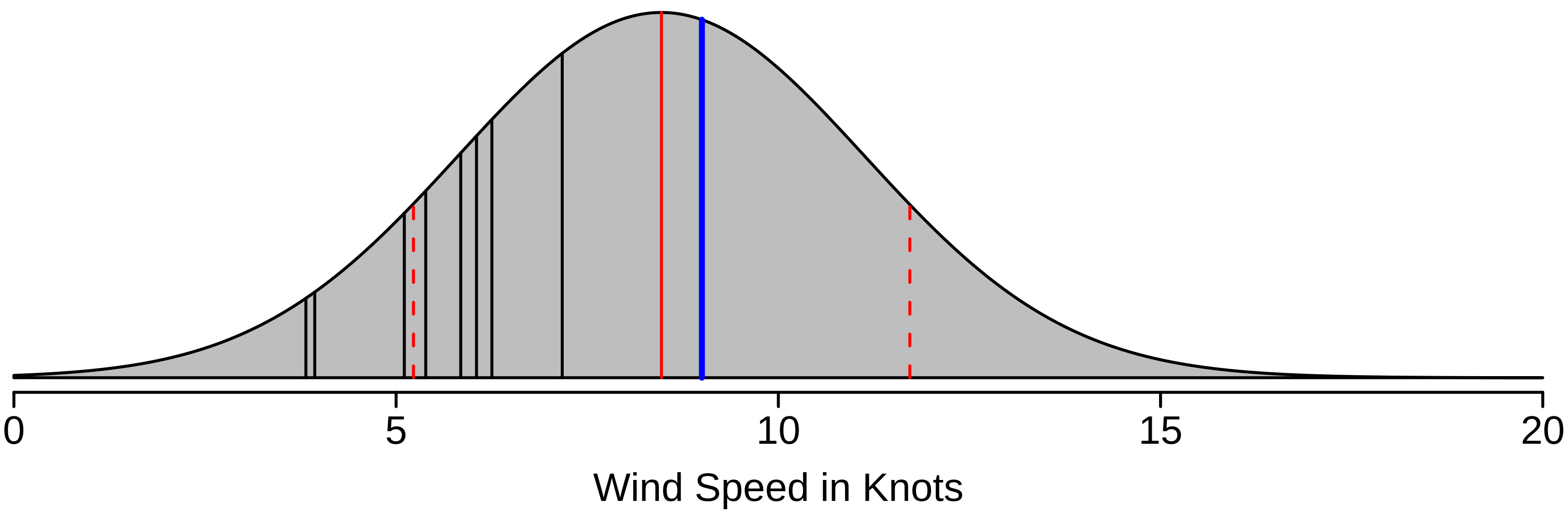
- **regional EMOS/NGR**: training data from all stations used, a single set of parameters, 20-day training period

- **local EMOS/NGR**: training data from station at hand only, one set of parameters for each station, 40-day training period

# Example: EMOS/NGR for wind speed

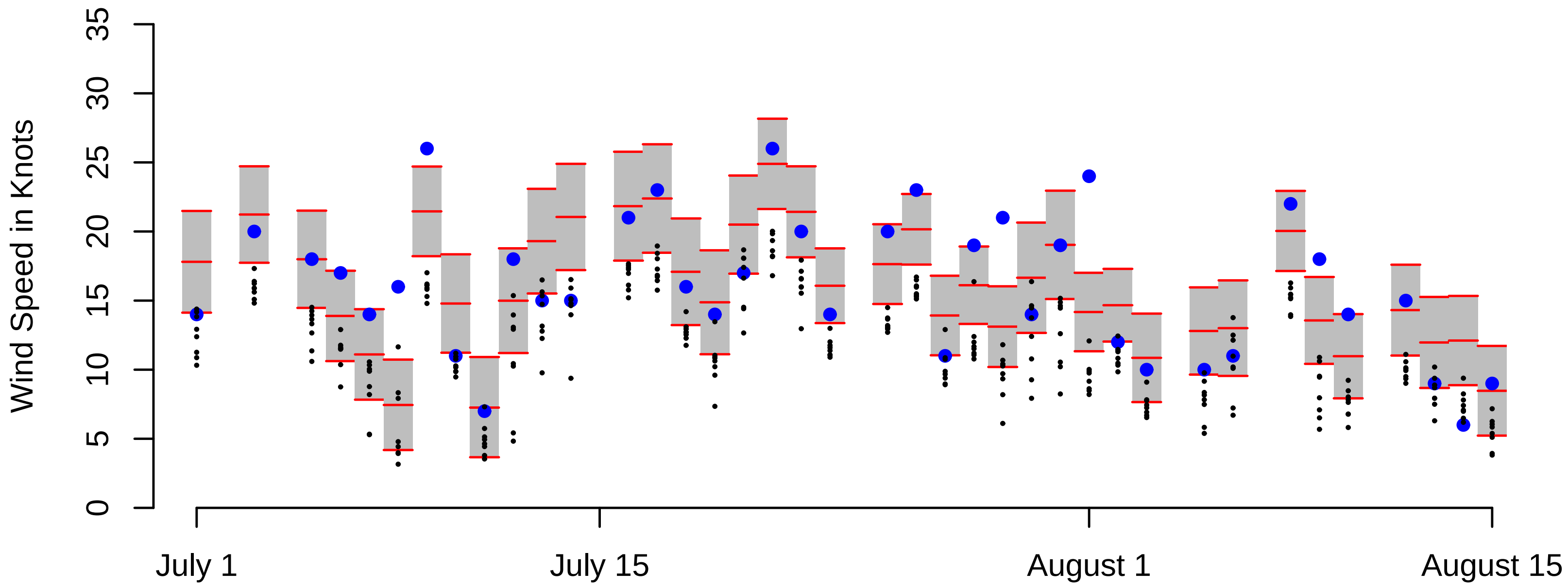
48-hour **local EMOS** postprocessed **predictive PDF** valid August 15, 2008 at **The Dalles, Oregon**

| | AVN | CMC | ETA | GSP | JMA | NGP | TCW | UKM | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 5.11 | 5.85 | 6.25 | 3.82 | 3.94 | 6.05 | 7.17 | 5.39 | | |
| $c_0$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ | $c_7$ | $c_8$ | $d_0$ | $d_1$ |
| 3.12 | 0.00 | 0.12 | 0.31 | 0.05 | 0.64 | 0.00 | 0.00 | 0.00 | 7.08 | 0.00 |



# Example: EMOS/NGR for wind speed

48-hour **local EMOS** postprocessed **predictive PDFs** at **The Dalles, Oregon**




**predictive performance** in calendar year 2008

| | MAE | CRPS |
|---|---|---|
| UWME | 4.42 | 3.53 |
| Regional EMOS/NGR | 3.96 | 2.85 |
| Local EMOS/NGR | 3.65 | 2.61 |

# Shortcomings of postprocessed forecasts

BMA and EMOS/NGR apply to a **single weather variable** at a **single location** and a **single look-ahead time** only

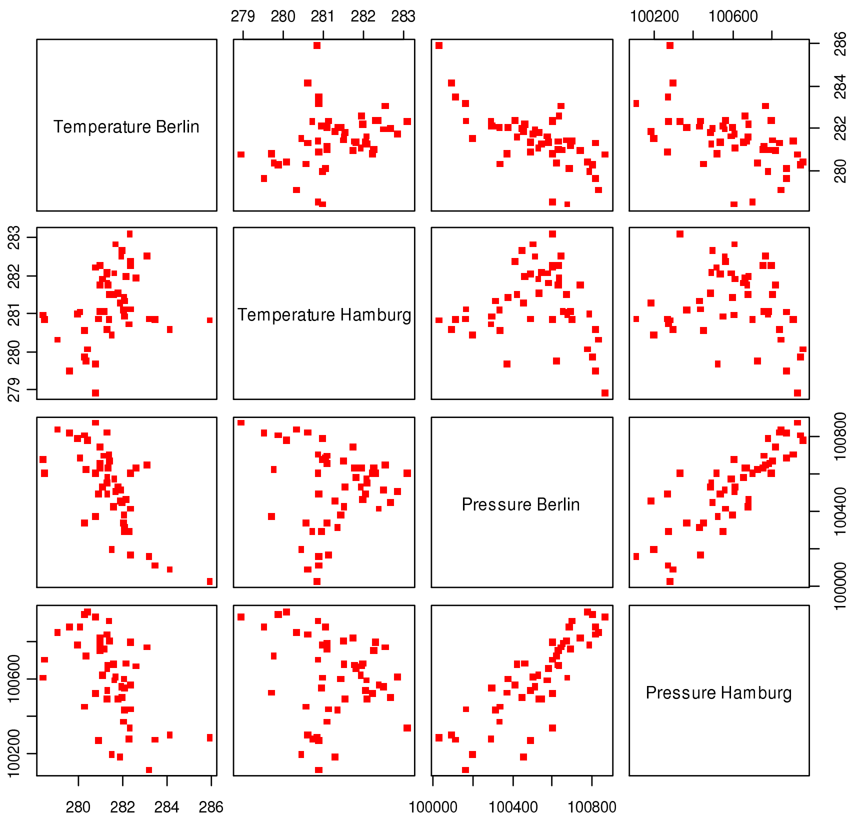
thus, the postprocessed PDFs fail to account for **multivariate dependence** structures

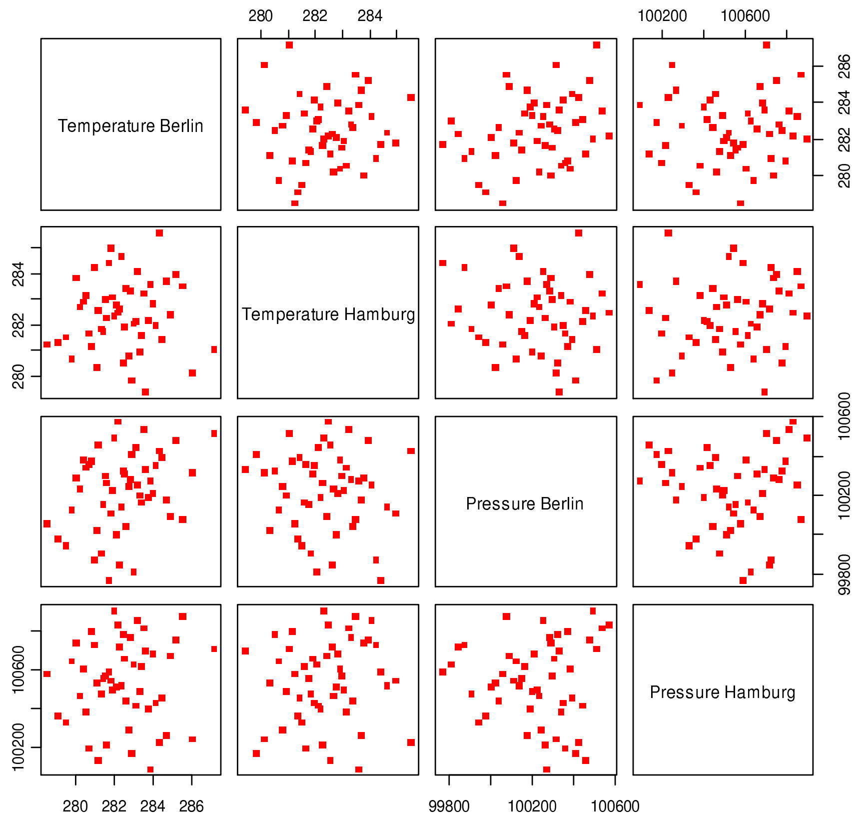
a pressing need now is to develop methods that yield **physically realistic** and **consistent** postprocessed ensemble forecasts of **spatio-temporal weather trajectories**

- for **multiple weather variables** at **multiple locations** and **multiple look-ahead times**
- in potentially very **high dimensions**: NWP models have **millions** of outputs
- key applications include **air traffic control** (SESAR), **ship routeing** and **flood management**

# Example: ECMWF ensemble

24-hour **ECMWF ensemble** forecast of **temperature** and **pressure** at Berlin and Hamburg valid May 27, 2010 **before** and **after** BMA **postprocessing**



## Theoretical background: Sklar's theorem

BMA and EMOS/NGR apply to a **single weather variable** at a **single location** and a **single look-ahead time** only

yielding a **univariate** postprocessed **predictive PDF**, $F_l$, for the univariate **weather quantity**, $Y_l$, where $l = 1, \ldots, L$

with each multi-index $l = (i, j, k)$ referring to **weather variable** $i$, **location** $j$ and **look-ahead time** $k$

we seek a **physically realistic** and consistent **multivariate** or **joint** predictive **PDF**, $F$, with margin $F_l$ for each $l = 1, \ldots, L$

**Sklar's theorem (1959):** every multivariate PDF $F$ with margins $F_1, \ldots, F_L$ can be written as

$$F(y_1, \ldots, y_L) = C(F_1(y_1), \ldots, F_L(y_L))$$

where $C : [0,1]^L \to [0,1]$ is a **copula**, i.e., a multivariate PDF with standard **uniform margins**

## Ensemble copula coupling

in order to issue **physically realistic** and **consistent** postprocessed ensemble forecasts of **spatio-temporal weather trajectories**

it remains to specify and fit a suitable **copula** $C : [0,1]^L \to [0,1]$, as reviewed by Schölzel and Friederichs (2008)

if the dimension $L$ is small, or if specific structure can be exploited, **parametric** familes of copulas work well

- Gel et al. (2004), Möller et al. (2012) and Schuhen et al. (2012) use **Gaussian** copulas
- parametric or semi-parametric alternatives include **elliptical**, **Archimedean**, hierarchical Archimedean and **pair** copulas

if $L$ is huge and no specific structure can be exploited, we need to resort to **non-parametric** approaches, with **ensemble copula coupling (ECC)** being a particularly attractive option

origins of ECC lie in the work of Clark et al. (2004), Bremnes (2007) and Krzysztofowicz and Toth (2008) and a conversation with Tom Hamill (2009)

## Ensemble copula coupling

given an NWP **ensemble** with $M$ members for the weather quantity $Y_l$, where $l = 1, \ldots, L$, **ECC** proceeds in **three steps**

**univariate postprocessing:** for each $l = 1, \ldots, L$ obtain a postprocessed **predictive PDF**, $F_l$

**quantization:** for each $l = 1, \ldots, L$, obtain a discrete **sample** of size $M$ from $F_l$

> **ECC-P**: ensemble mapping approach of **P**inson (2012)
>
> **ECC-Q**: use $M$ **equally spaced** **Q**uantiles of $F_l$ at the levels $(2m - 1)/(2m)$, where $m = 1, \ldots, M$
>
> **ECC-R**: draw a **R**andom **sample** of size $M$ from $F_l$

**ensemble reordering:** take the function $C : [0,1]^L \to [0,1]$ in Sklar's theorem to be the **empirical copula** of the **ensemble**, and apply it to the postprocessed ensemble

# Ensemble copula coupling

the postprocessed **ECC** ensemble retains the **flow-dependent** multivariate **rank dependence** structure in the NWP **ensemble**

it does so in the same way that the **Schaake shuffle** (Clark et al. 2004) adopts a dependence structure from **observational data**

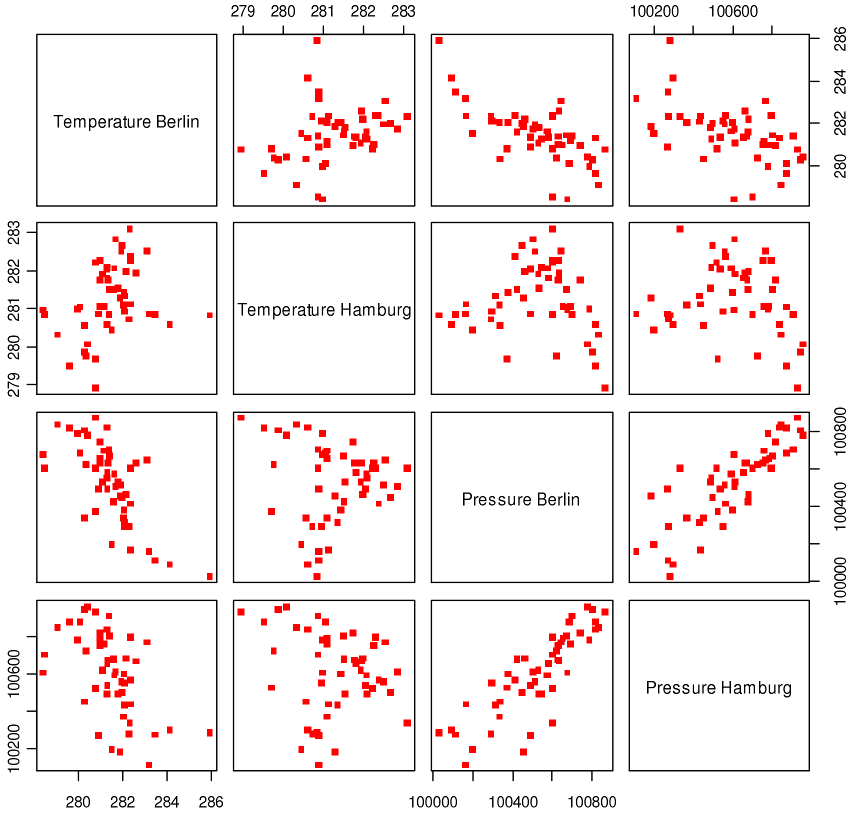
proposed by Bremnes (2007), Krzysztofowicz and Toth (2008) and in yesterday's talk by Jonathan Flowerdew, who explains the **ensemble reordering** very nicely (Flowerdew 2012, p. 17):

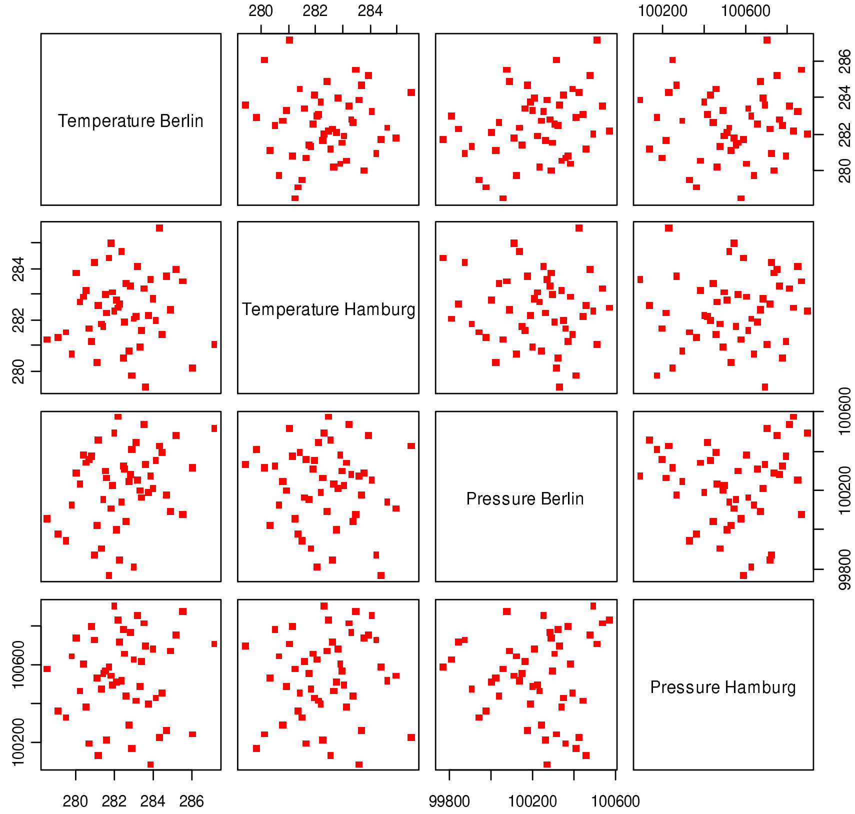
> "The key to preserving spatial, temporal and inter-variable structure is how this set of values is distributed among ensemble members. One can always construct ensemble members by sampling from the calibrated PDF, but this alone would produce spatially noisy fields lacking the correct correlations. Instead, *the values are assigned to ensemble members in the same order as the values from the raw ensemble:* the member with the locally highest rainfall remains locally highest, but with a calibrated rainfall magnitude."

for formulas, the copula connection and verification results, see poster 87 by Roman Schefzik

# Example: ECMWF ensemble

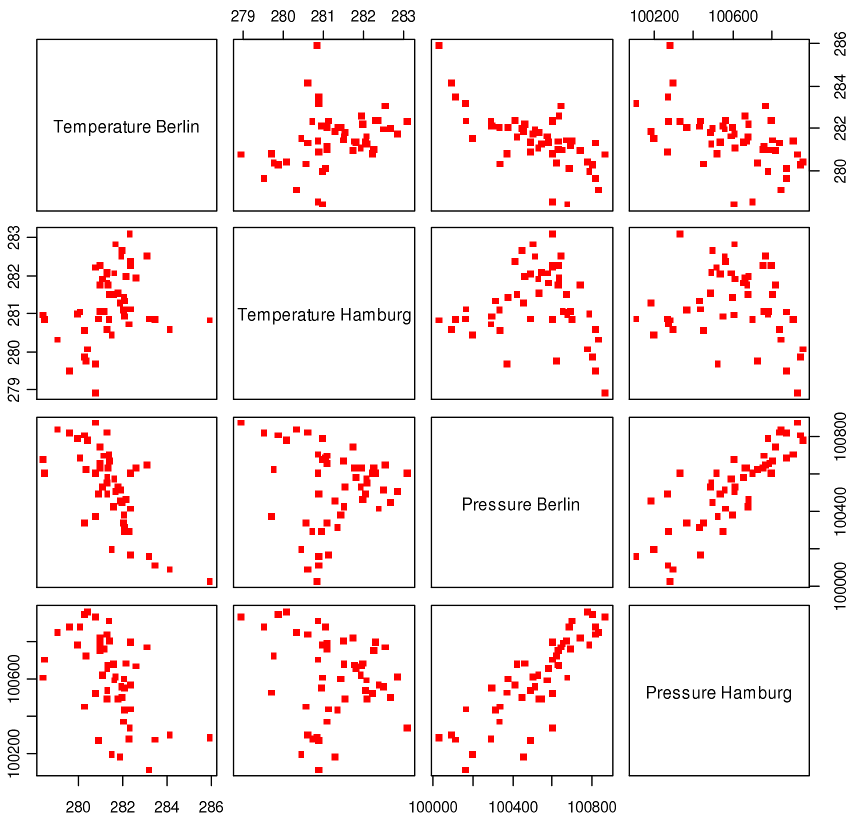
24-hour **ECMWF ensemble** forecast of **temperature** and **pressure** at Berlin and Hamburg valid May 27, 2010 **before** and **after** BMA **postprocessing**

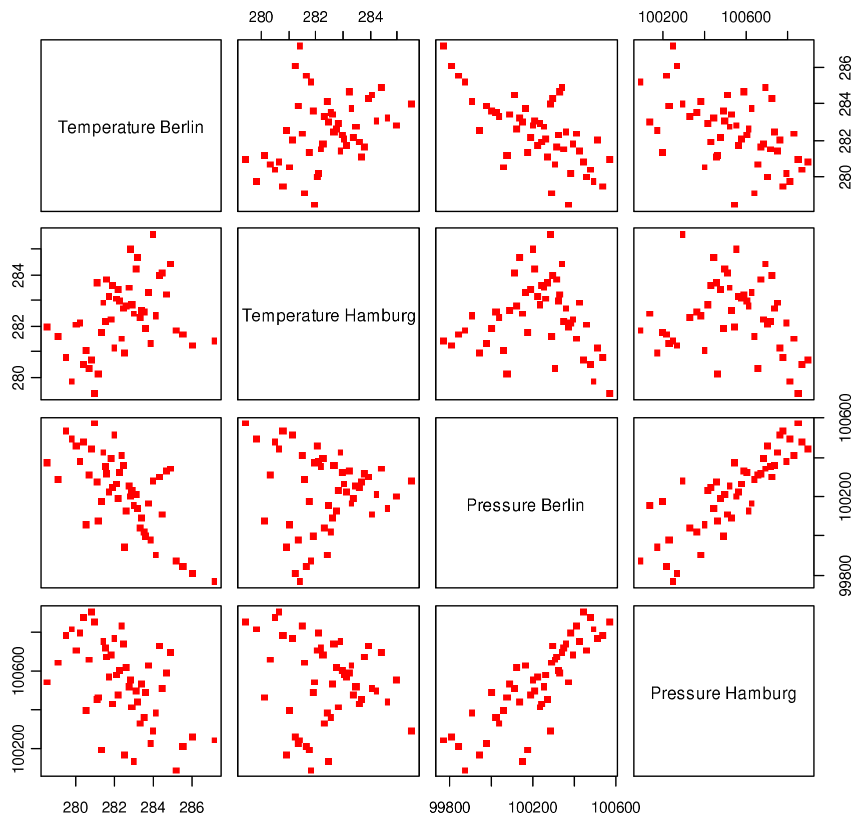


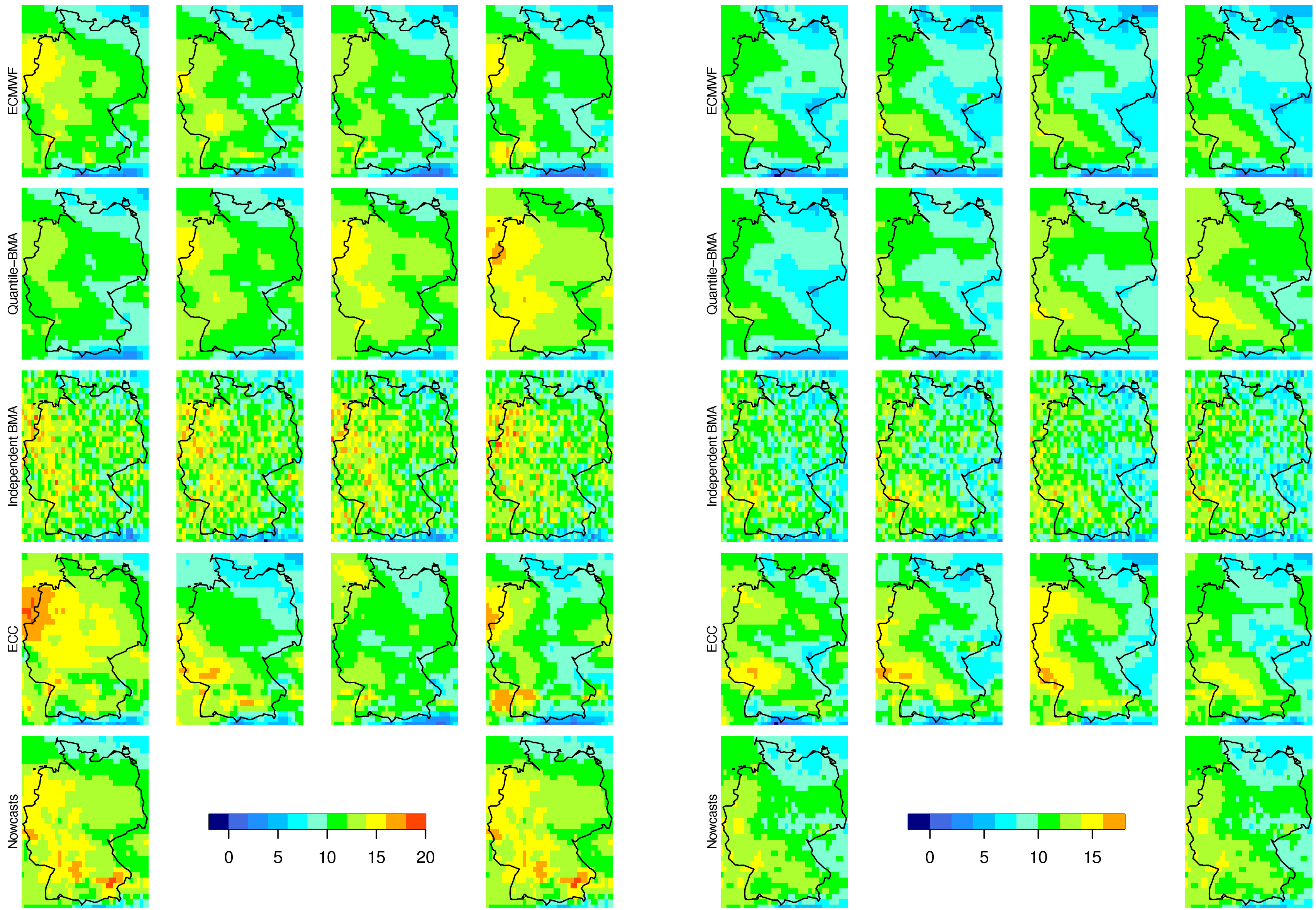
# Example: ECMWF ensemble

24-hour **ECMWF ensemble** forecast of **temperature** and **pressure** at Berlin and Hamburg valid May 27, 2010 **before** and **after** BMA **postprocessing** and **ECC**



# 24-h ECMWF EPS 2-m temperature 24–25 Aug 2011



## Discussion: Ensemble copula coupling (ECC)

the term **ensemble copula coupling (ECC)** refers to a general **three-stage** approach to ensemble postprocessing

**univariate postprocessing:** apply state of the art **statistical postprocessing** techniques to obtain calibrated and sharp **univariate** predictive **PDFs**

**quantization:** **sample** from the postprocessed univariate predictive PDFs, to obtain a **statistical ensemble** of the same size as the raw ensemble

**ensemble reordering:** **merge** the univariate statistical ensembles into **multivariate** spatio-temporal **weather trajectories**, by using the **empirical copula** of the original ensemble

# Discussion: Ensemble copula coupling (ECC)

the **ECC** ensemble retains the **flow-dependent** multivariate **rank dependence** structure in the raw **ensemble**

some **limitations**

- the ECC ensemble is constrained to have the same (small) number of members as the raw ensemble

- **perfect model assumption** with respect to the rank **dependence structure**, which frequently is in need of postprocesss-ing, too (poster 85, Martin Leutbecher)

- this need is addressed by Möller et al. (2012) and Schuhen et al. (2012), who apply a **Gaussian copula** approach to obtain calibrated probabilistic forecasts of **multivariate weather quantities**, such as wind vectors (poster 52, Isabel Albers)

## Discussion: Ensemble copula coupling (ECC)

the **ECC** ensemble retains the **flow-dependent** multivariate **rank dependence** structure in the raw **ensemble**

lots of **opportunities**

- theoretical backing by **Sklar's theorem**
- works with **any** univariate postprocessing technique, such as **EMOS/NGR**, **BMA** or **analog** methods (talk by Luca delle Monache; poster 19, Georg Mayr; poster 20, Tony Eckel)
- the ECC approach is **straightforward** to explain, **understand**, and **implement**
- **free lunch** — **negligible computational effort**, as compared to running an NWP ensemble or statistical postprocessing

**very broadly applicable** to yield physically consistent, calibrated and sharp **ensemble forecasts** of spatio-temporal **weather trajectories**

# 24-h ECMWF EPS 2-m temperature 24–25 Aug 2011