Control of the chemostat model

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MODEMIC
Treating and valorize biomass within a biorefinery context
An « explosion » of –omic data : a problematic of « big data »

High computation capacity

A (very) important deficit in modeling eduction within education in biology

How available data and modeling may help in better understanding and controlling bioprocesses?
Microbial ecosystems and the chemostat…

Bioprocess modeling

Observing and controlling the chemostat: a robust approach

An example: control of the anaerobic digestion process
Microbial ecosystems and the chemostat
Les écosystèmes microbiens
The chemostat

\[ S \xrightarrow{\mu(\cdot)} k_1 X + k_2 P \]

« Chemostat »

\[ S, X \]

\[ Q_{\text{in}} \]

\[ V, S, X \]

\[ Q_{\text{out}} \]

\[ S, X \]
The inventors of the chemostat

High diversity of scales
How using microbial ecosystems

Studying the functional heart of any bioprocess: a natural « complex » ecosystem
An immense world...

You, your cat and your red fish...
Conclusions

- Microbial ecosystems: complex systems;
- Used in a very high number of production/treatment fields;
- The chemostat is only 60 years old;
- Important research effort...
Lots of model!

Deterministic continuous dynamical models vs stochastic IBM

Campillo, 2011
Which model: the viewpoint of microbiologist…

Usually, they only know the two extreme cases: EDO-based and IBM (used for describing space)!

Then use of a model instead of another depends essentially of their collaboration and of their own modeling culture!
Which model: the viewpoint of modelists/mathematicians...

Such nonlinear models may be usefull for my research to be applied...
How modeling an ideal bioprocess?

Until recently...
How modeling an ideal bioprocess?

Modern biological tools reveal rather...
The chemostat model

The associated continuous deterministic model

\[ \frac{d(SV)}{dt} = \text{Input mass of } S - \text{Output mass of } S \ldots \]

\[ \ldots + \text{production} - \text{consumption} \]

\[
\begin{cases}
\frac{dS}{dt} = S_{in} \frac{Q}{V} - S \frac{Q}{V} - \frac{\mu(S)}{Y} X = (S_{in} - S)D - \frac{\mu(S)}{Y} X \\
\frac{dX}{dt} = \mu(S)X - \frac{Q}{V} X = (\mu(S) - D) X
\end{cases}
\]
The chemostat model

- Kinetic modeling

\[
\begin{align*}
\dot{s} &= -\mu(s)x + D(s_{in} - s) \\
\dot{x} &= \mu(s)x - Dx
\end{align*}
\]

\[
\mu(s) = \frac{\bar{\mu}s}{K + s + s^2/K_i}
\]
The chemostat model

- Matching data: identifying yields and kinetics coefficients

![Chemostat model diagram]
Conclusions

- Important research on chemostat modeling;
- To study microorganism growth (ecological questions);
- To optimize its functioning...
Observing and controlling the chemostat…
Chemostat with inhibition

\[
\begin{align*}
\frac{dS}{dt} &= (S_{in} - S)D - k\mu(S)X \\
\frac{dX}{dt} &= (\mu(S) - D)X \\
\mu(S) &= \frac{\bar{\mu}S}{S + K_S + \frac{S^2}{K_I}} \\
\end{align*}
\]

\[
\bar{\mu} = 0.045, \quad K_S = 10, \quad K_I = 10
\]
Qualitative properties of the model

The chemostat model

1 equilibrium: washout

\[ D > \max_{s \in [0, S_{in}]} \mu(s) \]

3 equilibria: bistability

\[ \mu(s_{in}) < D < \max_{s \in [0, S_{in}]} \mu(s) \]

2 equilibria: 1 stable and one unstable (washout)
The chemostat model

- State space representation

Cas #2: Under the condition

\[ \mu(s_{\text{in}}) < D < \max_{s \in [0, S_{\text{in}}]} \mu(s) \]
Closing the loop

Controlling the chemostat

Diagram:

- **D** → **Chemostat** → **Control** → **Controled chemostat**
- **S** → **Chemostat** → **Control** → **Controled chemostat**
- **S_c** → **Control** → **Controled chemostat**

Symbols:
- **D**: Dilution rate
- **S**: Substrate concentration
- **S_c**: Controled substrate concentration
- **Chemostat**: Continuous culture system
- **Control**: Control system
- **Controled chemostat**: Controlled chemostat system
Stabilizing the chemostat

Solution: Output Feedback

Example #1: Linearizing feedback control

\[ D(s, x) = \frac{\mu(s)x - \lambda(s - s^*)}{s_{in} - s} \]

Cf. Bastin et Dochain 1986

Example #2: PI Controller

\[ D(s) = G_1(s - s^*) + G_2 \int_0^t (s(\tau) - s^*)d\tau \]

Cf. Alvarez Lopez-Arenas 2012 (and many others)

Example #N: ...
Stabilizing the chemostat

Solution: Output Feedback

*Exemple #1*: Linearizing feedback control

\[ D(s, x) = \frac{\mu(s)x - \lambda(s - s^*)}{s_{in} - s} \]

*Drawback*: We must know everything! \((\mu + \sin, s \text{ and } x...)\)

Assuming we only know \(s\) and we have uncertainty on others input and parameters, how stabilizing the system with guaranteed performances?
Stabilizing the chemostat

Solution: A robust observer scheme

\[
\begin{aligned}
\dot{x} &= f(x, u) \\
y &= g(x)
\end{aligned}
\]

\[
\begin{aligned}
\dot{x} &= f_{\text{obs}}(\hat{x}, u, y) \\
\hat{y} &= g_{\text{obs}}(\hat{x})
\end{aligned}
\]

\[
\lim_{t \to +\infty} (x(t) - \hat{x}(t)) = 0
\]
Stabilizing the chemostat

Solution: An interesting change of variables

\[
d\hat{Z}(t) = \left(S_{in} - \hat{Z}(t)\right)D(t)
\]
\[
\hat{Z}(0) = \hat{Z}_0
\]
\[
\hat{X}(t) = Y\left(\hat{Z}(t) - S(t)\right)
\]

\[
\lim_{t \to +\infty} Z(t) = S_{in}
\]
Stabilizing the chemostat

- Robustifying the asymptotic observer with respect to uncertainty (inputs + measurements + parameters)

\[
\begin{align*}
\frac{d\hat{Z}^-}{dt} &= \left(S^-_{in} - \hat{Z}^-(t)\right)D(t) \\
\hat{Z}^-(0) &= \hat{Z}^-_0 \\
\hat{X}^-(t) &= Y\left(\hat{Z}^-(t) - S(t)\right)
\end{align*}
\]

\[
\begin{align*}
\frac{d\hat{Z}^+}{dt} &= \left(S^+_{in} - \hat{Z}^+(t)\right)D(t) \\
\hat{Z}^+(0) &= \hat{Z}^+_0 \\
\hat{X}^+(t) &= Y\left(\hat{Z}^+(t) - S(t)\right)
\end{align*}
\]

\[
\lim_{t \to +\infty} Z^-(t) = S^-_{in}
\]

\[
\lim_{t \to +\infty} Z^+(t) = S^+_{in}
\]
An exemple : controlling the anaerobic digestion process
Stabilizing the chemostat

- Application

\[ S_1 \rightarrow X_1 + S_2 + CO_2 \]
\[ S_2 \rightarrow X_2 + CH_4 + CO_2 \]
The model

\[
\begin{align*}
\dot{X}_1 &= (\mu_1 - \alpha D)X_1 \\
\dot{X}_2 &= (\mu_2 - \alpha D)X_2 \\
\dot{Z} &= D(Z^i - Z) \\
\dot{S}_1 &= D(S_1^i - S_1) - k_1\mu_1 X_1 \\
\dot{S}_2 &= D(S_2^i - S_2) + k_2\mu_1 X_1 - k_3\mu_2 X_2 \\
\dot{C}_{TI} &= D(C^i_{TI} - C_{TI}) + k_4\mu_1 X_1 + k_5\mu_2 X_2 - Q_{CO_2}
\end{align*}
\]
Stabilizing the chemostat

- The robust observer: the inputs

Graphs showing the outputs of the chemostat model over time, with data points and trend lines.
Stabilizing the chemostat

- The robust observer: the measurements
Stabilizing the chemostat

- The robust estimations
Stabilizing the chemostat

- Coupling the robust observer with the robust control design: application to the anaerobic digestion

\[
D(t) = \text{sat}_{[0,D]} \left( \frac{k\mu^{+/-}(t)\hat{X}^{+/-}(t) - \lambda(S(t) - S^*)}{S^{+/-}(t) - S(t)} \right)
\]
Stabilizing the chemostat

Results: control and controled variable
Conclusions and perspectives
Conclusions and perspectives

- Model of the chemostat
- Stabilizing robustly the chemostat
- Towards biocontrol approaches...
Thank you for your attention!