Control of the chemostat model

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Le LBE-INRA et MODEMIC

- INRA
- About 80 people

Treating and valorize biomass within a bioreffinery context

CONTEXT

- An « explosion » of –omic data : a problematic of « big data »
- High computation capacity
- A (very) important deficit in modeling eduction within education in biology
- How available data and modeling may help in better understanding and controling bioproceses?

CONTENT

- Microbial ecosystems and the chemostat…
- Bioprocess modeling
- Observing and controling the chemostat : a robust approach
- An example : control of the anaerobic digestion process

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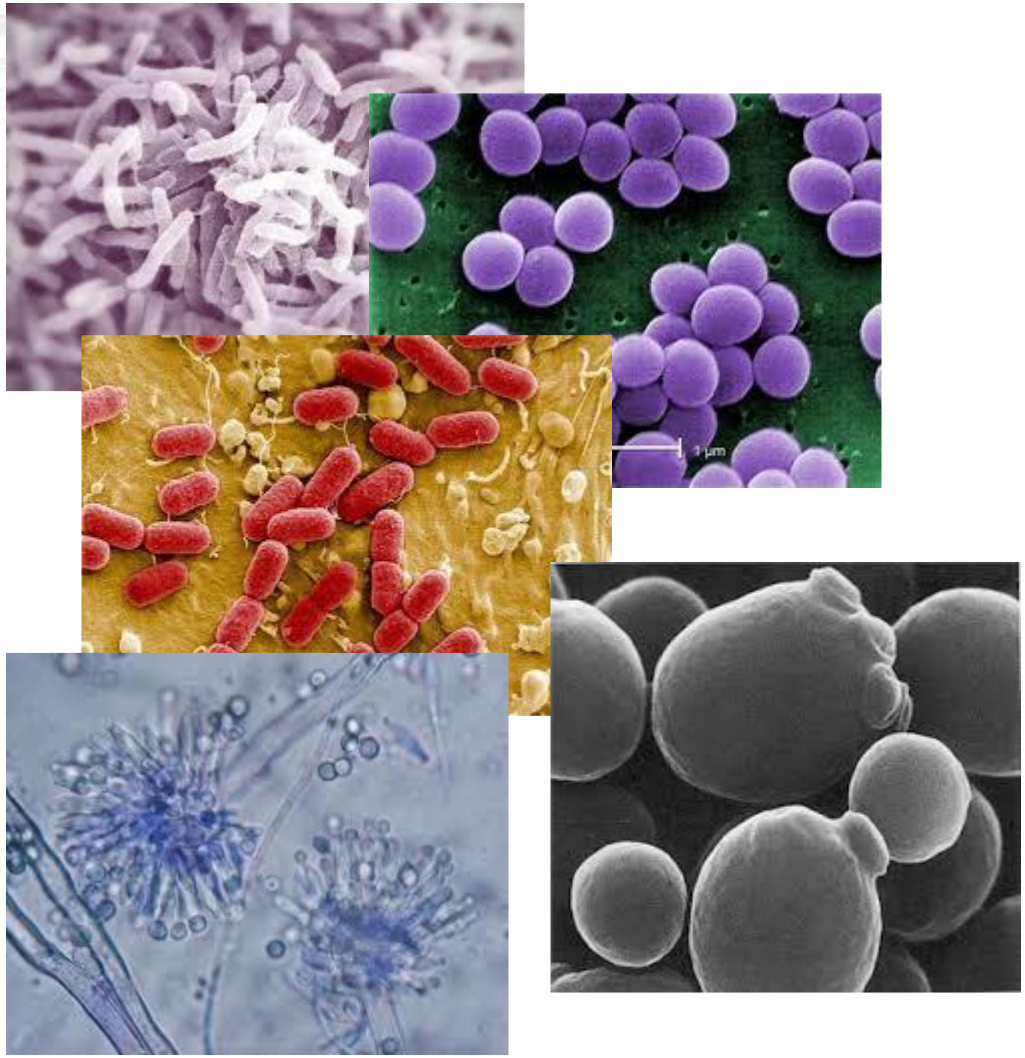
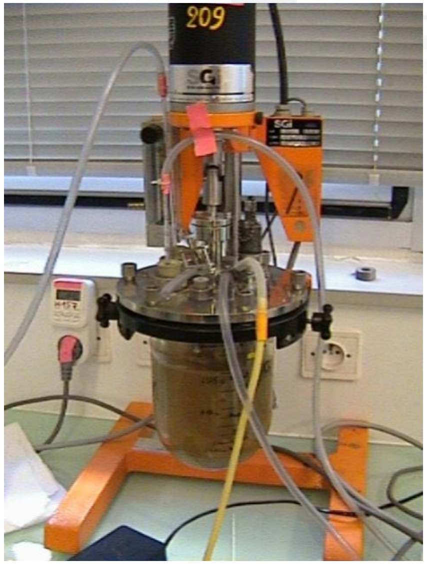
Microbial ecosystems and the chemostat

Les écosystèmes microbiens

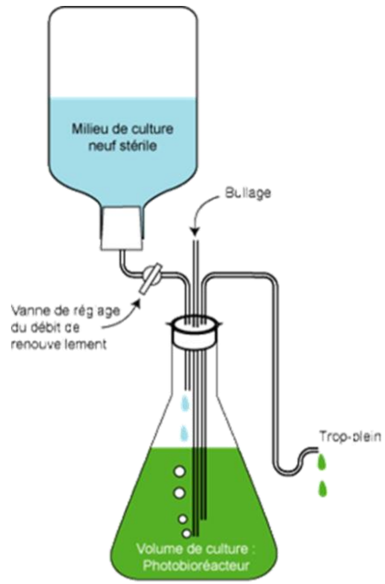


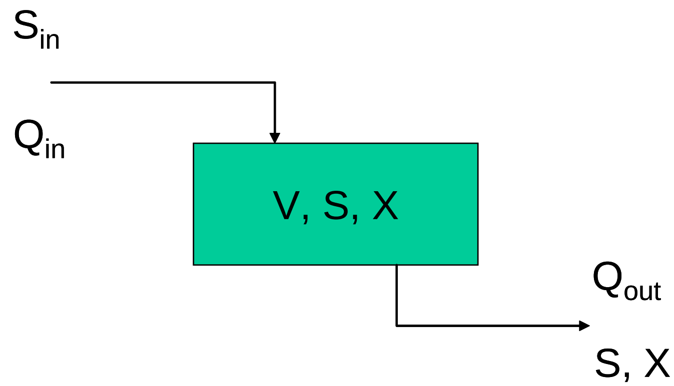
The chemostat



« Chemostat »

$$S \xrightarrow{\mu(.)} k_1 X + k_2 P$$



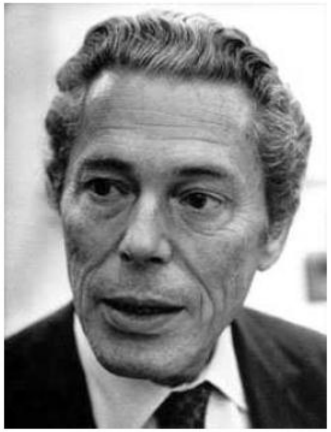


Le chemostat

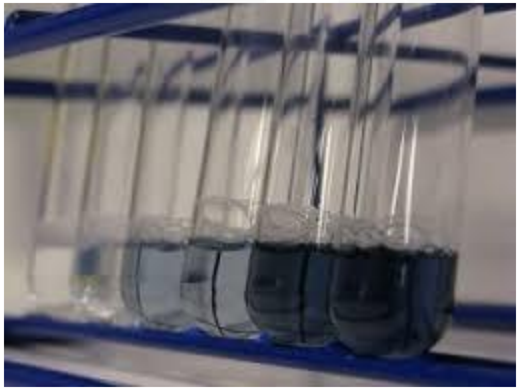
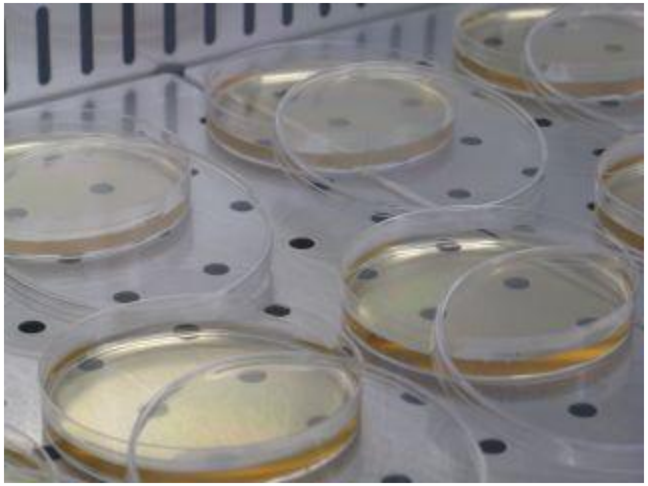
The inventors of the chemostat

- Novick A. and Szilard L. (1950), *Description of the chemostat*. Science, 112, 715-716
- Monod, J., *La technique de culture continue theorie et applications*. Ann. Inst. Pasteur, 79, 390-410, 1950



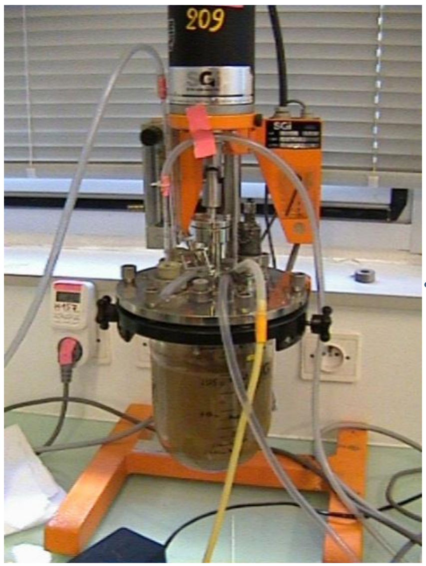


High diversity of scales



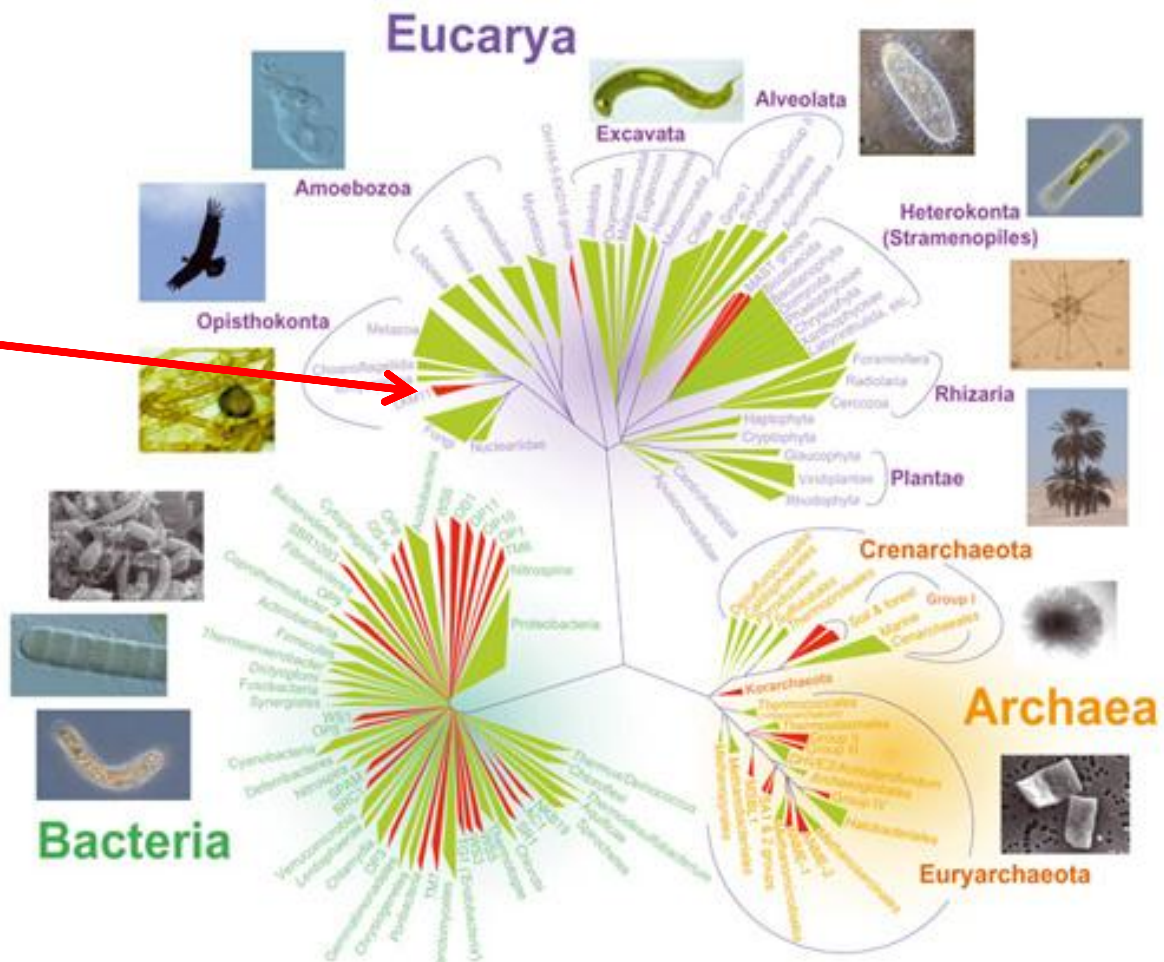
How using microbial ecosystems



Studying the functional heart of any bioprocess: a natural « complex » ecosystem

An immense world…

You, your cat and your red fish…



Conclusions

- Microbial ecosystems : complex systems;
- Used in a very high number of production/treatment fields
- The chemostat is only 60 years old;
- Important research effort…

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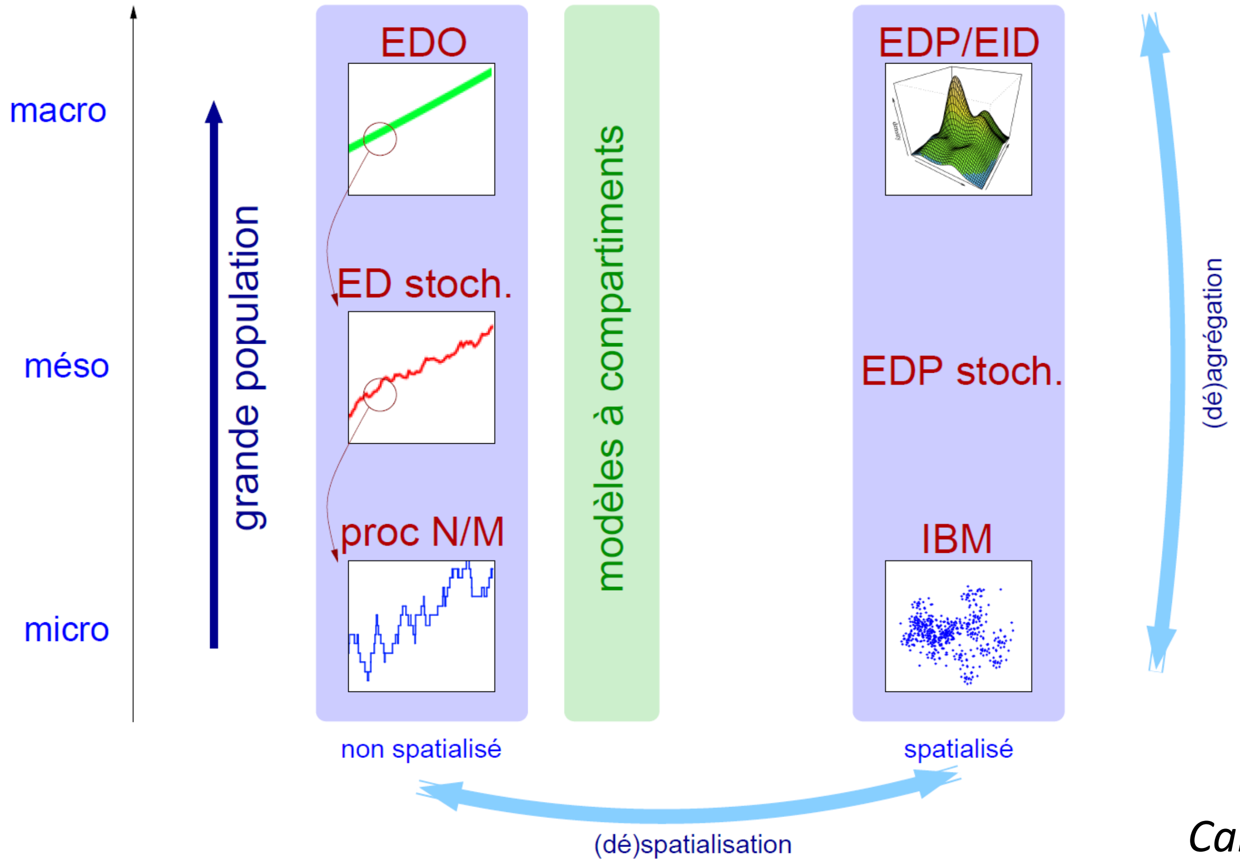
Modeling

Lots of model!

Deterministic continuous dynamical models vs stochastic IBM



Campillo, 2011

Which model : the viewpoint of microbiologist…

Usually, they only know the two extreme cases : EDO-based and IBM (used for describing space)!

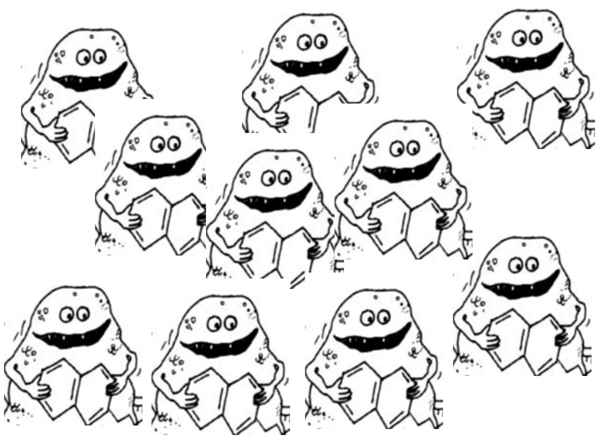
Thenuse of a model instead of another depends essentially of their collaboration and of their own modeling culture!

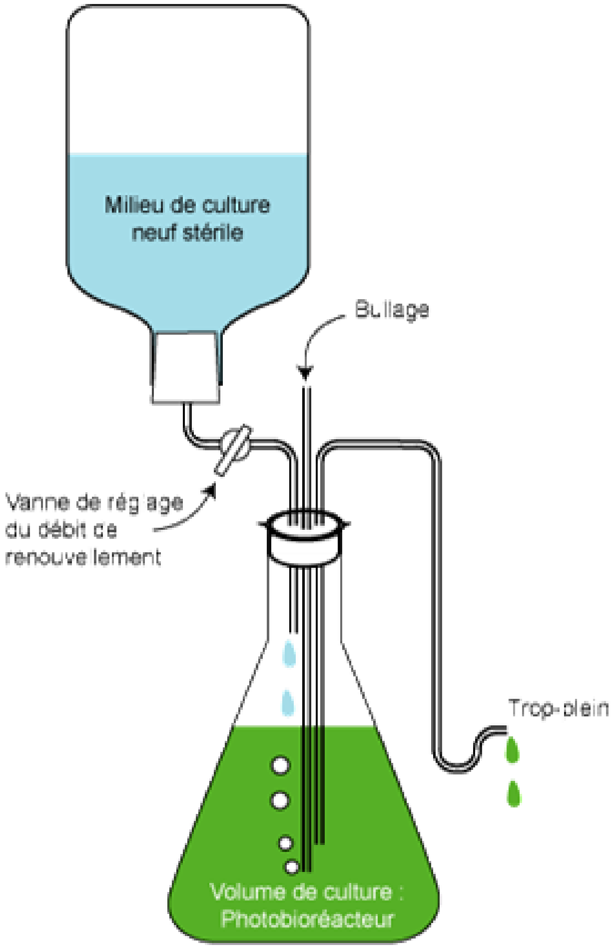
Which model : the viewpoint of modelists/mathematicians…

Such nonlinear models may be usefull for my research to be applied…

How modling an ideal bioprocess?

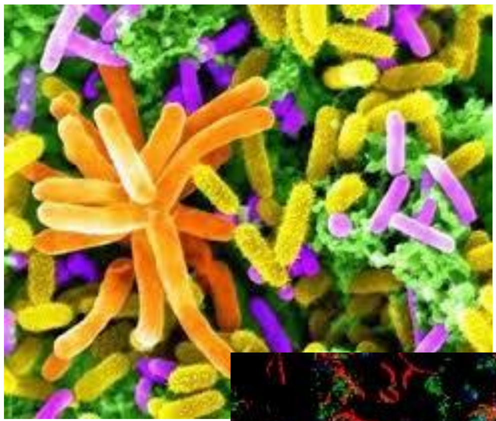
- ❑ Until recently…

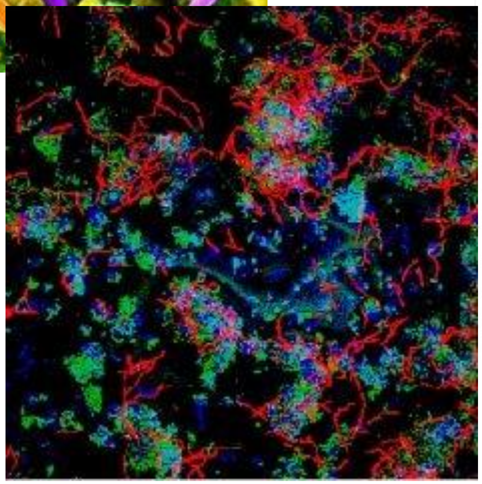


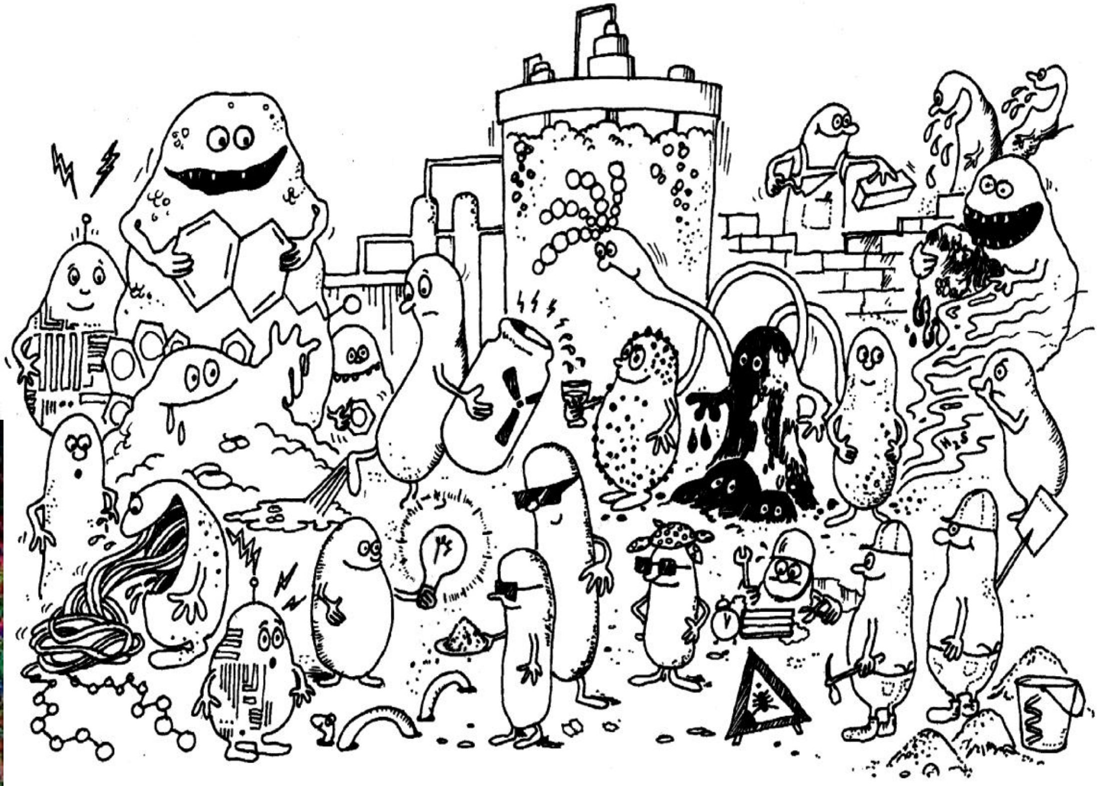


How modeling an ideal bioprocess?

- Modern biological tools reveal rather…







The chemostat model

- The associated continuous deterministic model

$$\frac{d(SV)}{dt} = \text{"Input mass of S"} - \text{"Ouput mass of S"} \ldots$$

$$\ldots + \text{"production"} - \text{"consumption"}$$

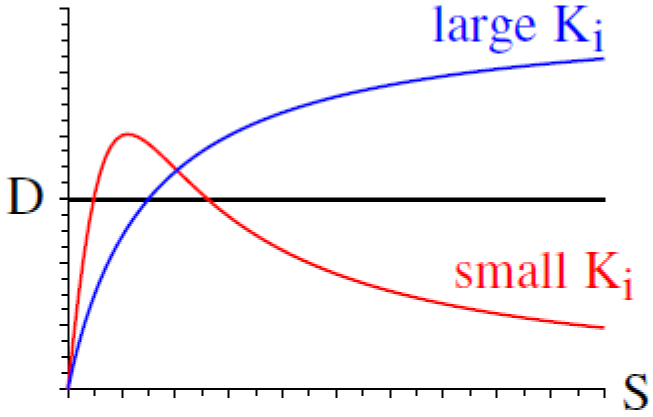
$$\begin{cases} \dfrac{dS}{dt} = S_{in}\dfrac{Q}{V} - S\dfrac{Q}{V} - \dfrac{\mu(S)}{Y}X = (S_{in} - S)D - \dfrac{\mu(S)}{Y}X \\ \dfrac{dX}{dt} = \mu(S)X - \dfrac{Q}{V}X = (\mu(S) - D)X \end{cases}$$

The chemostat model

- Kinetic modeling

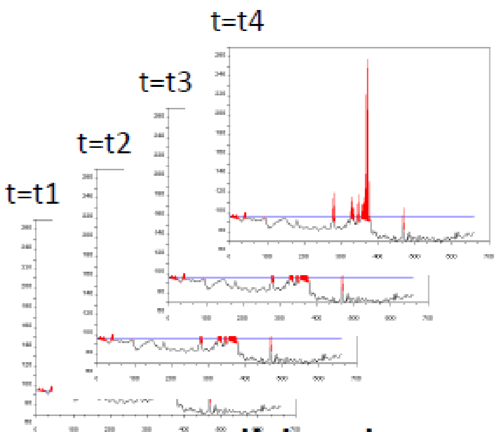
$$\begin{aligned} \dot{s} &= -\mu(s)x + D(s_{in} - s) \\ \dot{x} &= \mu(s)x - Dx \end{aligned}$$

$$\boxed{\mu(s) = \frac{\bar{\mu}s}{K + s + s^2/K_i}}$$

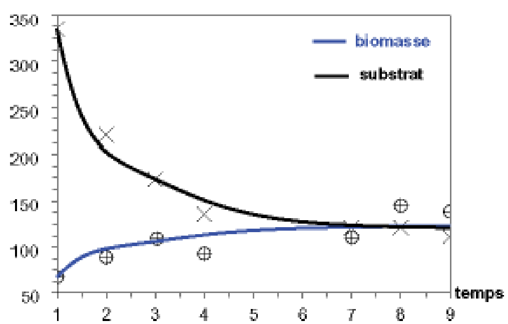


The chemostat model

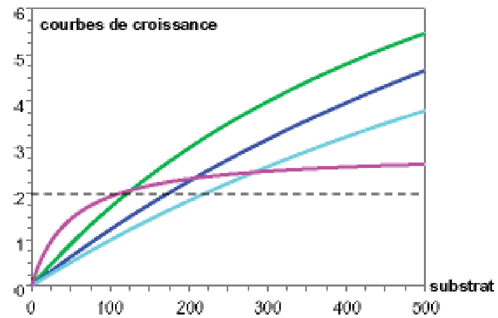
- Matching data : identifying yields and kinetics coefficients



spectres d'abondances



mesures macroscopiques



modèle fonctionnel

Conclusions

- Important research on chemostat modeling;
- To study microorganism growth (ecological questions);
- To optimize its functioning…

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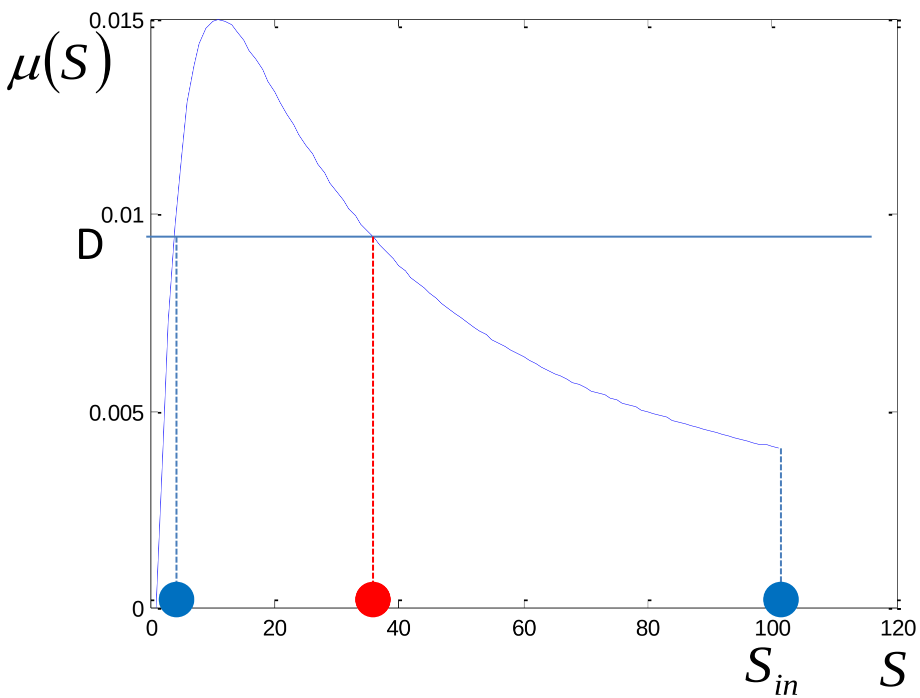
Observing and controling the chemostat…

The chemostat model

- Chemostat with inhibition

$$\begin{cases} \dfrac{dS}{dt} = (S_{in} - S)D - k\mu(S)X \\ \dfrac{dX}{dt} = (\mu(S) - D)X \end{cases}$$

$$\mu(S) = \frac{\bar{\mu} S}{S + K_S + \dfrac{S^2}{K_I}}$$



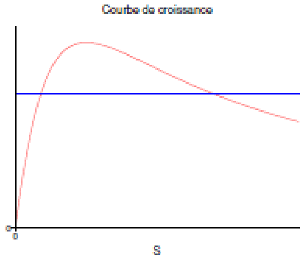
$$\bar{\mu} = 0.045, K_S = 10, \ K_I = 10$$

The chemostat model

❑ Qualitative propserties of the model

$$D > \max_{s \in [0, S_{in}]} \mu(s)$$

1 equilibrium : washout



$$\mu(s_{in}) < D < \max_{s \in [0, S_{in}]} \mu(s)$$

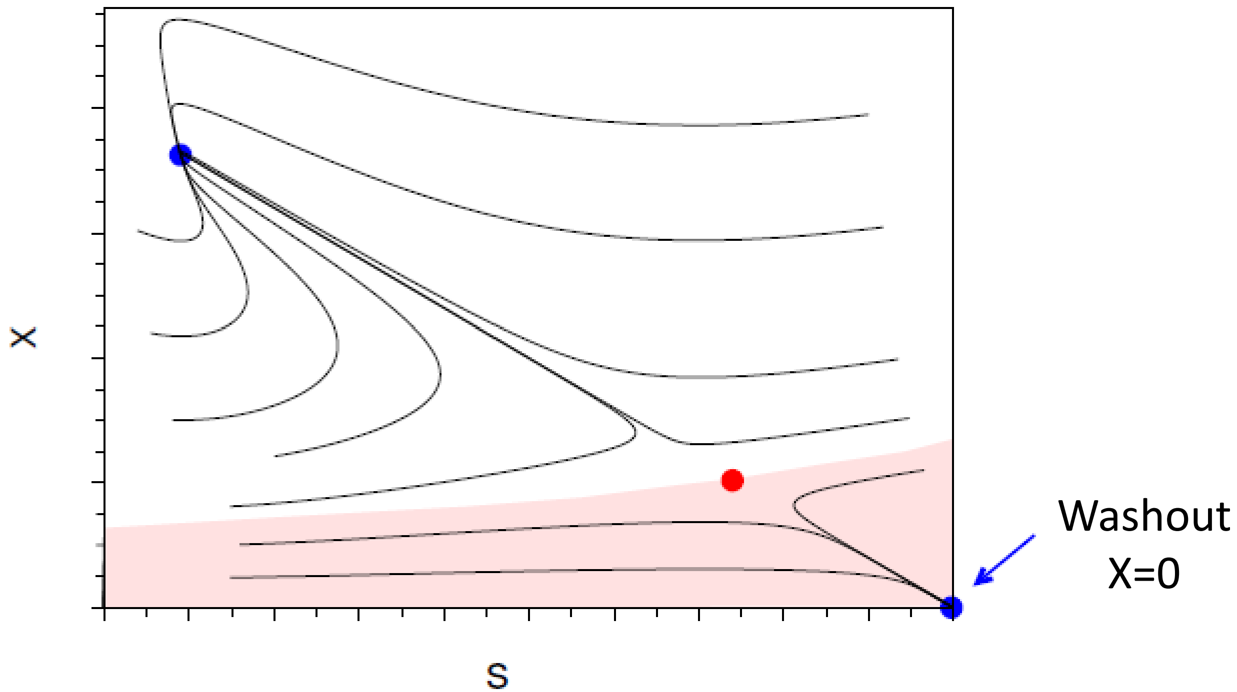
3 equilibria : bistability

$$D < \mu(s_{in})$$

2 equilibria : 1 stable and one unstable (washout)

The chemostat model

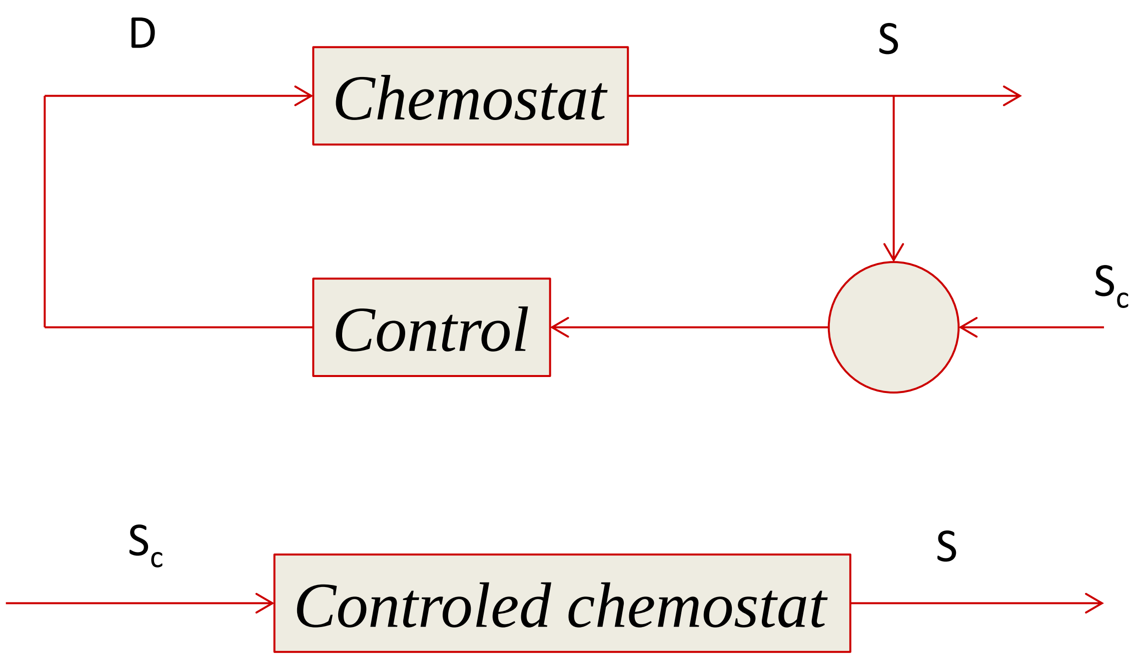
- State space representation



Cas #2 : Under the condition $\mu(s_{in}) < D < \max_{s \in [0, S_{in}]} \mu(s)$

Controling the chemostat

- Closing the loop



Stabilizing the chemostat

- Solution : Output Feedback

Example #1 : Linearizing feedback control

$$D(s,x) = \frac{\mu(s)x - \lambda(s - s^\star)}{s_{in} - s}$$

Cf. Bastin et Dochain 1986

Example #2 : PI Controller

$$D(s) = G_1(s - s^\star) + G_2 \int_0^t (s(\tau) - s^\star) d\tau$$

Cf. Alvarez Lopez-Arenas 2012 (and many others)

Example #N : …

Stabilizing the chemostat

- Solution : Output Feedback

Exemple #1 : Linearizing feedback control

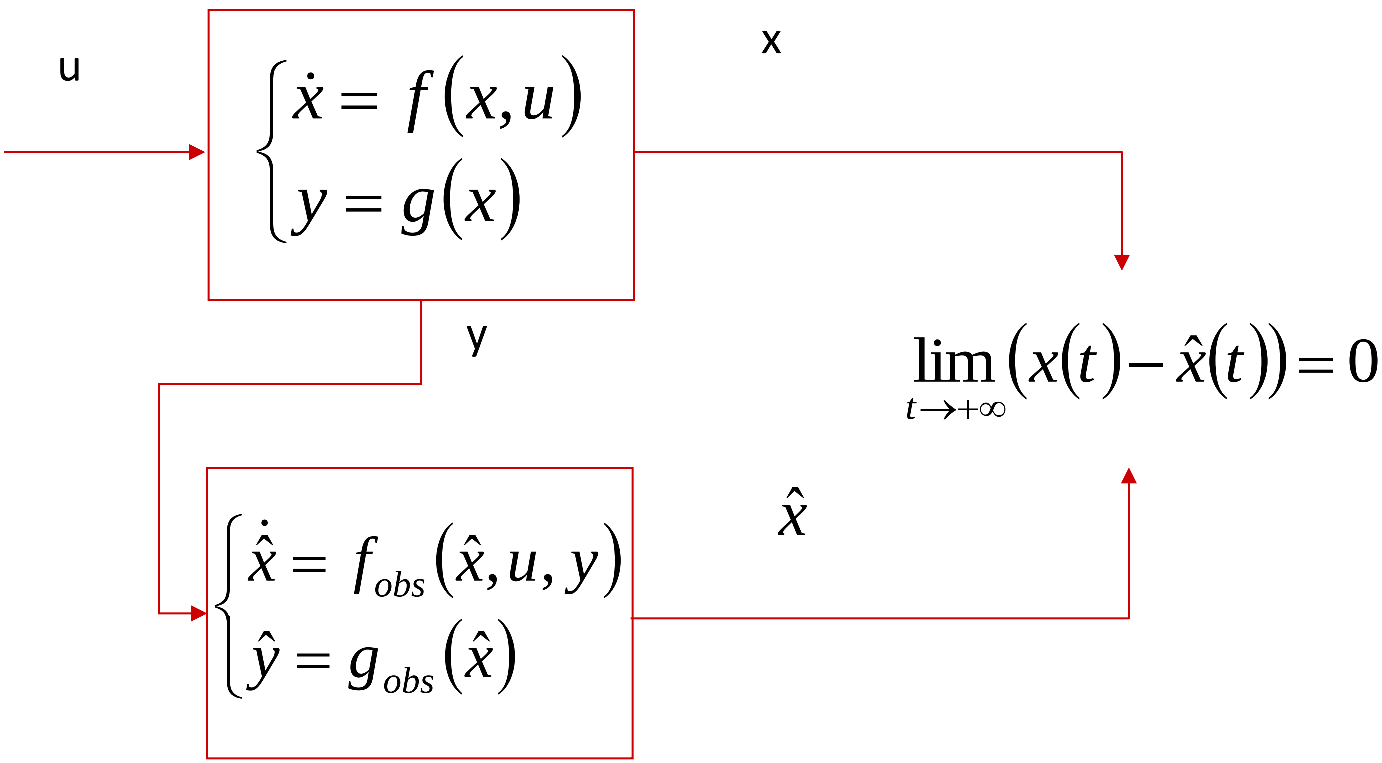
$$D(s,x) = \frac{\mu(s)x - \lambda(s - s^\star)}{s_{in} - s}$$

Drawback : We must know everything! (mu + sin, s and x…)

Assuming we only know s and we have uncertainty on others input and parameters, how stabilizing the system with guaranteed performances?

Stabilizing the chemostat

- Solution : A robust observer scheme



Stabilizing the chemostat

- Solution : An interesting change of variables

$$\begin{cases} \dfrac{d\hat{Z}(t)}{dt} = \left(S_{in} - \hat{Z}(t)\right)D(t) \\ \hat{Z}(0) = \hat{Z}_0 \\ \hat{X}(t) = Y\left(\hat{Z}(t) - S(t)\right) \end{cases}$$

$$\lim_{t \to +\infty} Z(t) = S_{in}$$

Stabilizing the chemostat

- Robustifying the asymptotic observer with respect to uncertainty (inputs + measurements + parameters)

$$\begin{cases} \dfrac{d\hat{Z}^-(t)}{dt} = \left(S_{in}^- - \hat{Z}^-(t)\right)D(t) \\ \hat{Z}^-(0) = \hat{Z}_0^- \\ \hat{X}^-(t) = Y\left(\hat{Z}^-(t) - S(t)\right) \end{cases}$$

$$\begin{cases} \dfrac{d\hat{Z}^+(t)}{dt} = \left(S_{in}^+ - \hat{Z}^+(t)\right)D(t) \\ \hat{Z}^+(0) = \hat{Z}_0^+ \\ \hat{X}^+(t) = Y\left(\hat{Z}^+(t) - S(t)\right) \end{cases}$$

$$\lim_{t\to+\infty} Z^-(t) = S_{in}^-$$

$$\lim_{t\to+\infty} Z^+(t) = S_{in}^+$$

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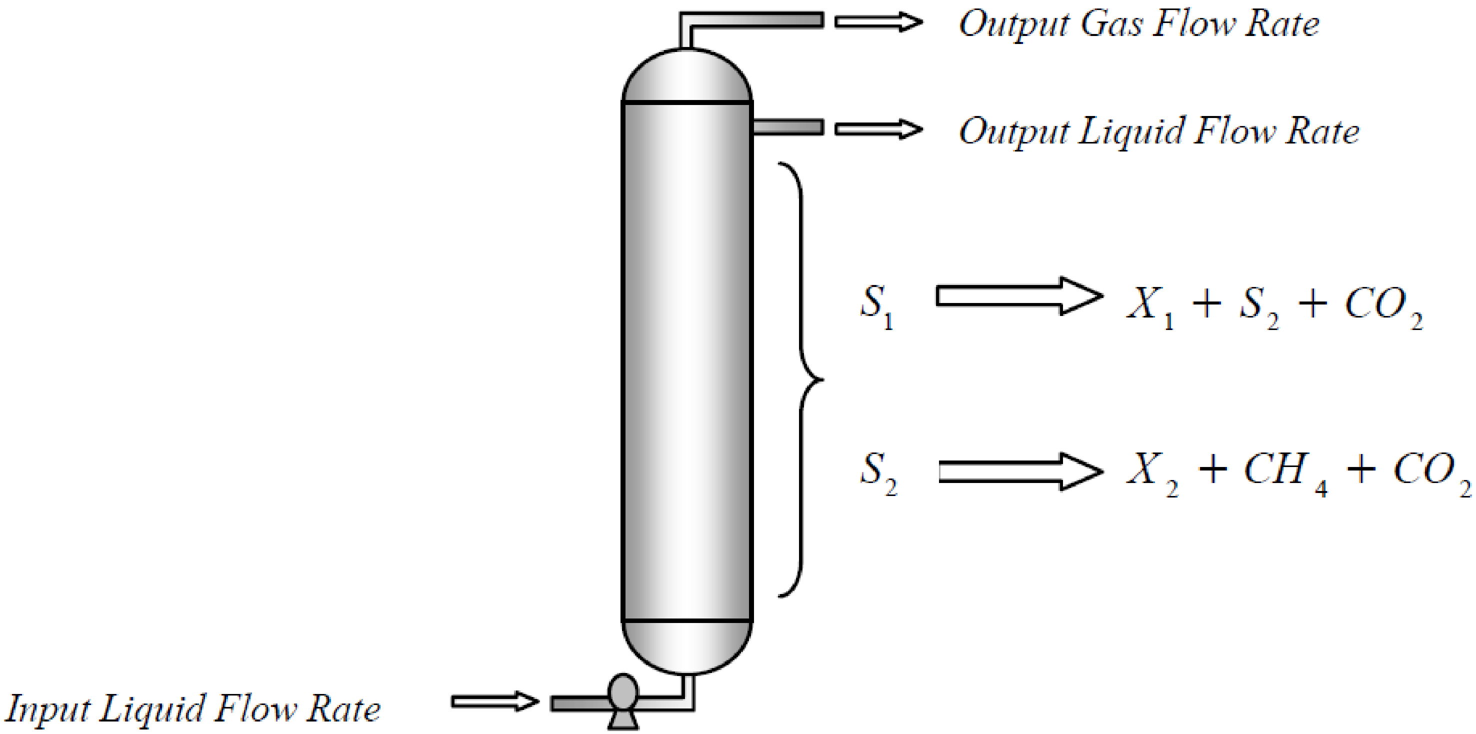
An exemple : controling the anaerobic digestion process

Stabilizing the chemostat

- Application



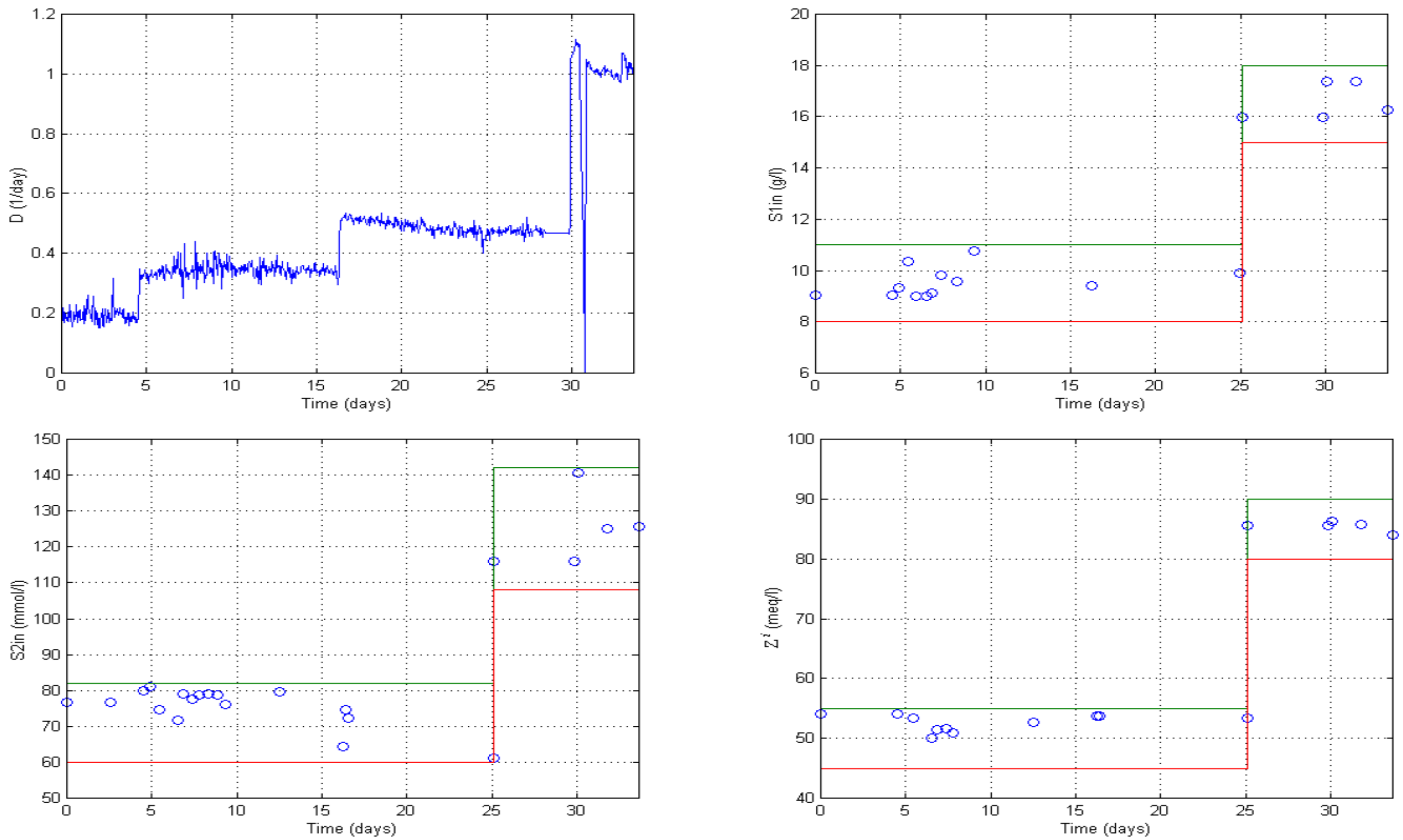
Stabilizing the chemostat

- The model

$$
\begin{cases}
\dot{X}_1 = (\mu_1 - \alpha D)X_1 \\
\dot{X}_2 = (\mu_2 - \alpha D)X_2 \\
\dot{Z} = D(Z^i - Z) \\
\dot{S}_1 = D(S_1^i - S_1) - k_1\mu_1 X_1 \\
\dot{S}_2 = D(S_2^i - S_2) + k_2\mu_1 X_1 - k_3\mu_2 X_2 \\
\dot{C}_{\mathrm{TI}} = D(C_{\mathrm{TI}}^i - C_{\mathrm{TI}}) + k_4\mu_1 X_1 + k_5\mu_2 X_2 - Q_{\mathrm{CO_2}}
\end{cases}
$$

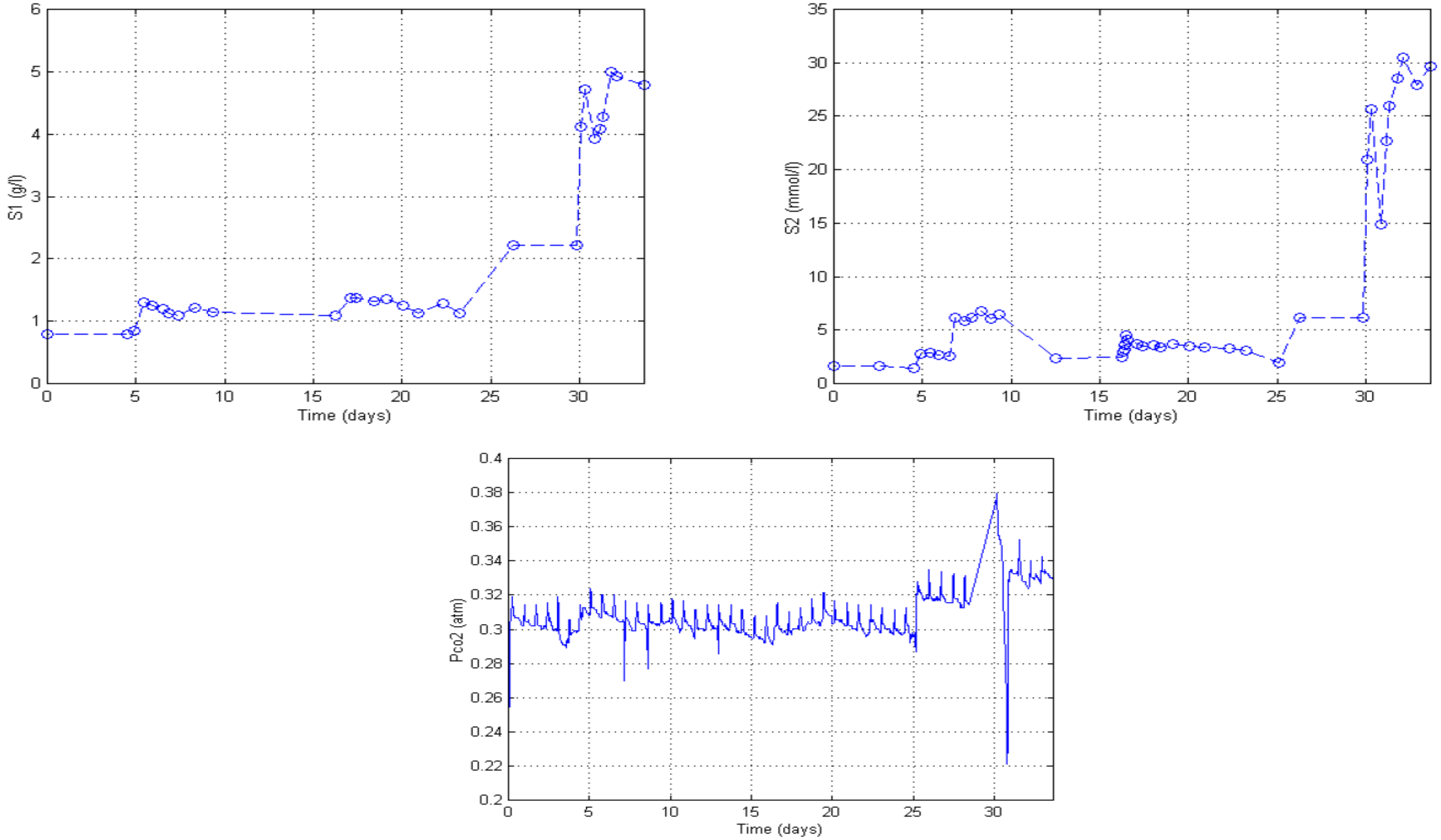
Stabilizing the chemostat

- The robust observer : the inputs



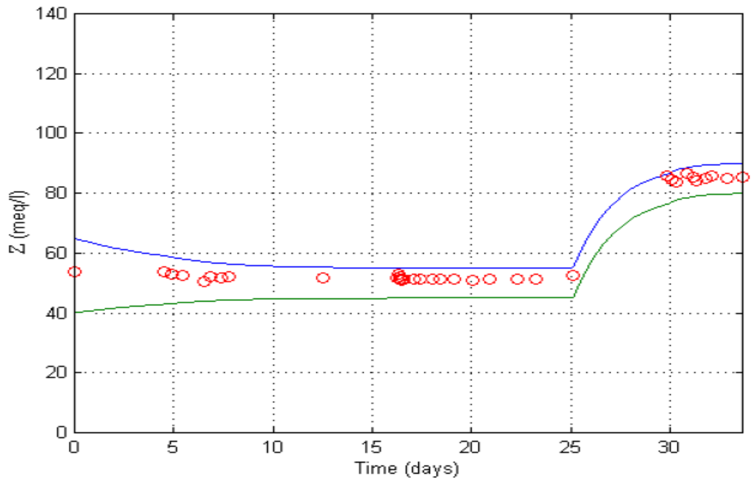
Stabilizing the chemostat

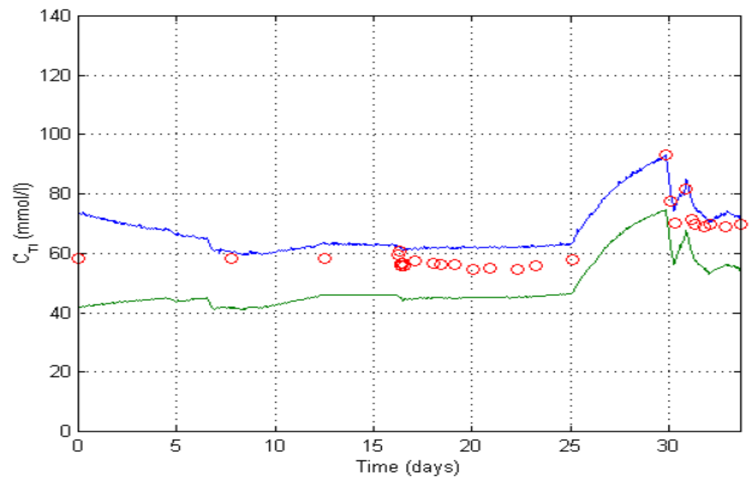
- The robust observer : the measurements



Stabilizing the chemostat

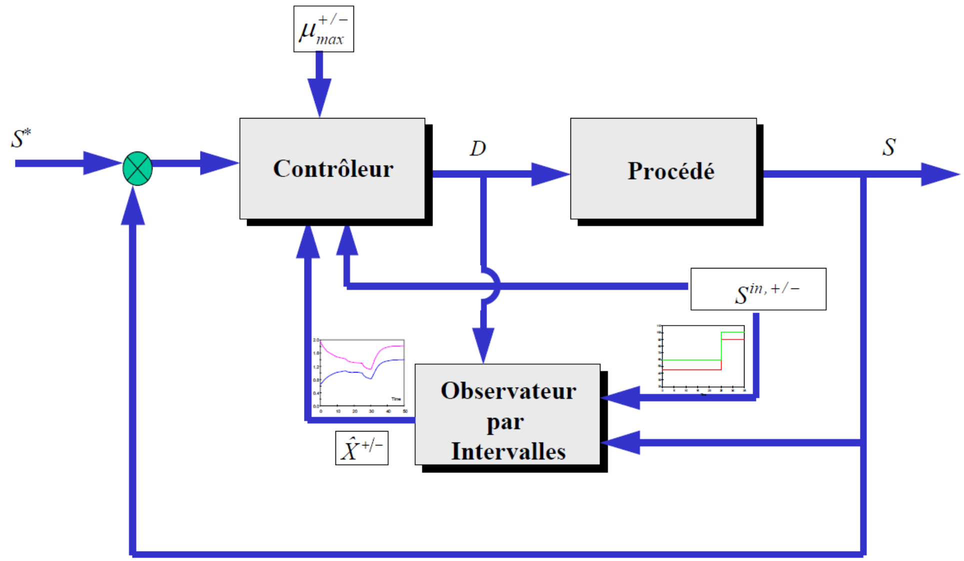
- The robust estimations





Stabilizing the chemostat

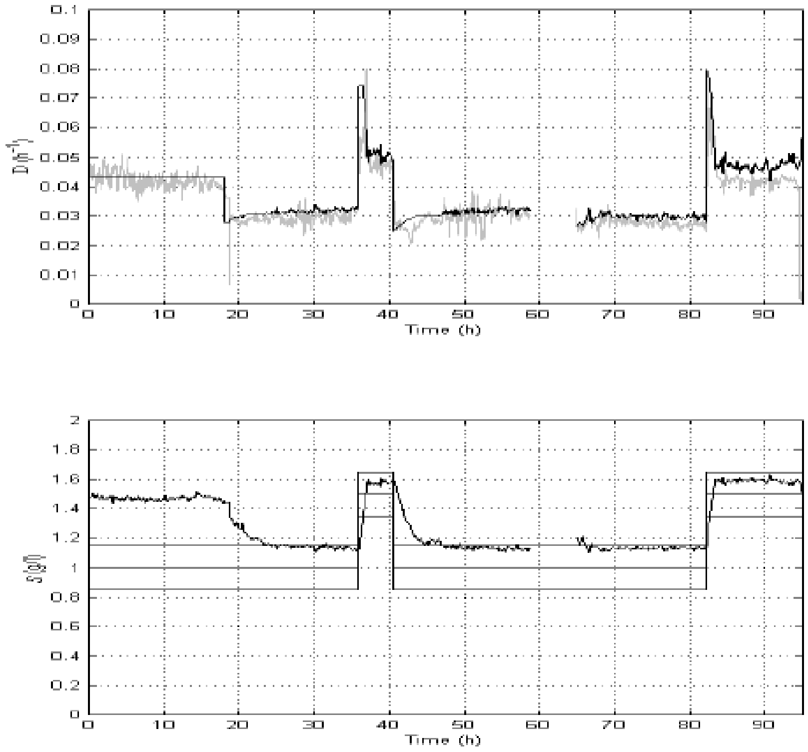
- Coupling the robust observer with the robust control design : application to the anaerobic digestion



$$D(t) = sat_{[0,\bar{D}]}\left(\frac{k\mu^{+/-}(t)\hat{X}^{+/-}(t) - \lambda\left(S(t) - S^*\right)}{S_{in}^{+/-}(t) - S(t)}\right)$$

Stabilizing the chemostat

- Results : control and controled variable



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Conclusions and perspectives

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Conclusions and perspectives

- Model of the chemostat
- Stabilizing robustly the chemostat
- Towards biocontrol approaches…

Thank you for your attention!