Comparing the SEKF with the DEnKF on a land surface model

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Aims and justification for study

- Simplified extended Kalman filter (SEKF, Mahfouf et al. (2009)) currently used within SURFEX framework to assimilate soil moisture observations in a land surface model;
- SEKF already improves soil moisture estimates compared with open loop (no assimilation);
- Ensemble Kalman filter (EnKF) may be a better DA method for the future;
- **Main aim of study is to compare the performance of the SEKF with an EnKF.**
- The two methods differ in the representation of the background-error covariance;
- We follow in the footsteps of similar work by Reichle and Koster (2003); Sabater et al. (2006); Mahfouf (2007).
Increase the domain size and model complexity as more knowledge is gained:

1. Single site (SABRES in-situ obs);
2. 12 in-situ observation sites (SMOSMANIA network);
3. France domain;
Land surface model: ISBA-Ags

- The surface layer ($WG1$) (top 1cm) is forced by precipitation and evaporation, and restored towards an equilibrium value ($w_{eq}$);
- $w_{eq}$ reached when gravity matches capillary forces;
- Forcing term causes rapid intermittent changes in $WG1$ (e.g. rainfall event), while restoration takes about a day.
- A deep layer $WG2$ (1-3m deep), called the 'root zone', exists below $WG1$;
- Root zone is also affected by vegetation transpiration and drainage;
- Changes in $WG2$ much slower than $WG1$, due to its larger depth;
- Includes vegetation dynamics (not important for this talk).
WG1 model and obs at Sabres site

Bias corrected (CDF matched) obs used for assimilation:

Figure 1: WG1 model and observations ($m^3/m^3$).
WG2 model and obs at Sabres site

Bias corrected (CDF matched) obs used for verification only:

Figure 2: WG2 model and observations ($m^3/m^3$).
Land data assimilation system - LDAS

- Atmospheric forcing provided by mesoscale analysis at 8km resolution (SAFRAN);
- **Optimal interpolation** with screen-level temp/humidity obs provides soil moisture analyses for France NWP model (AROME);
- **Simplified extended Kalman filter (SEKF)** assimilates daily WG1 satellite obs (ASCAT) and LAI obs (SPOT-VEG) over France;
- **SEKF** used for monitoring carbon and water fluxes;
- Our study compares the **SEKF** with the **Deterministic Ensemble Kalman filter (DEnKF, Sakov and Oke (2008))**.
Simplified Extended Kalman filter (SEKF)

- Background \( (\mathbf{x}^b(t_i)) \) is a nonlinear propagation of previous analysis:
  \[
  \mathbf{x}^b(t_i) = M(\mathbf{x}^a(t_{i-1}))
  \]  
  (1)

- Analysis \( (\mathbf{x}^a(t_i)) \) is calculated using one gridpoint observation;
- Assimilated observation \( (\mathbf{y}^o) \) is weighted using Kalman gain \( (\mathbf{K}) \):
  \[
  \mathbf{x}^a = \mathbf{x}^b + \mathbf{K}(\mathbf{y}^o - H(\mathbf{x}^b)),
  \]  
  (2)

  where
  \[
  \mathbf{K} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1},
  \]  
  (3)

  where \( \mathbf{B} \) is a climatological and diagonal background-error covariance.
SEKF observation operator

- Model predicted observation $y = H(x)$;
- Jacobian of observation operator for obs $k$ and model point $l$ calculated by finite differences:

$$H^{kl} = \frac{y^k}{\delta x^l} = \left[ \frac{y^k(x + \delta x^l) - y^k(x)}{\delta x^l} \right]$$

$$= \left[ \frac{H^k(M(x^b(t_i-1) + \delta x^l)) - H^k(M(x(t_i-1)))}{\delta x^l(t_i-1)} \right]; \quad (4)$$

- In operational setup each grid-cell is split into 12 land-cover types (not relevant for this talk).
**DEnKF**

- Background ensemble calculated from previous analysis ensemble:
  \[ x_j^b(t_i) = M(x_j^a(t_{i-1})) \text{, for } j = 1, \ldots, m. \]  
  \[(5)\]

- Background anomaly matrix \( X^b \) (of dimension \( n \times m \)) comes from \( m \) column vectors \( \delta x_j^b \):
  \[ X^b = \frac{1}{\sqrt{m-1}} \begin{bmatrix} x_1^b - \bar{x}^b & \cdots & x_m^b - \bar{x}^b \end{bmatrix}, \]
  \[(6)\]
  where \( m \) is the number of ensemble members.

- Ensemble background-error covariance:
  \[ P^b = X^b(X^b)^T. \]
  \[(7)\]

- DEnKF halves Kalman gain in analysis perturbation update:
  \[ X^a = X^b - \frac{1}{2} KHX^b. \]
  \[(8)\]

- Deterministic analysis comes from ensemble mean:
  \[ \bar{x}^a = \bar{x}^b + K(y^o - H(x^b)). \]
  \[(9)\]
Experimental setup

- Firstly test methods at single site (Sabres);
- Then test methods over 12 sites;
- Daily assimilated WG1 (surface soil moisture) observations;
- WG2 (root-zone soil moisture) observations used for verification purposes;
- 12 ensemble members for DEnKF;
- Period 2007-2010 (first year spin-up);
- Observation error std = 0.023 m³/m³ (half satellite observation error) estimated using Desrozier diagnostics (Desroziers *et al.*, 2005);
- Imperfect model - model error approximated in background-error covariance.
Background/Model error calibration

- SEKF $B$ variances tuned to produce smallest analysis errors;
- SEKF $B$ includes contribution from model error;
- DEnKF $P^b$ collapses without allowing for model error;
- DEnKF model error tuned using additive noise and perturbed precipitation forcing;
- Additive noise sampled from Gaussian distribution (Mitchell et al., 2002);
- Precipitation perturbations sampled from lognormal distribution (Mahfouf, 2007);
DEnKF model error calibration

- Time correlated noise $\phi$ introduced using 1st order auto-regressive model (Mahfouf, 2007):

$$
\phi(t_{i+1}) = \nu \phi(t_i) + \psi \sqrt{1 - \nu^2},
$$

$$
\nu = \frac{1}{1 + \Delta t / \tau},
$$

(10)

where $\psi$ is Gaussian white noise and $\tau$ is the temporal correlation.

- Additive noise prescribed to analysis ensemble members:

$$
x^a_j \leftarrow x^a_j + \phi_j
$$

(11)

- Precipitation ($Pr$) perturbed for each ensemble member $j$:

$$
Pr_{log} = \log(Pr + 1)
$$

$$
Pr'_j = \exp(Pr_{log} + \phi_j) - 1,
$$

(12)

where $Pr'_j$ is the perturbed precipitation for ensemble member $j$. 
## Important algorithmic differences

<table>
<thead>
<tr>
<th>Property</th>
<th>SEKF</th>
<th>DEnKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow-dependent background-error covariance</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Requires Jacobians of obs operator</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Stochastic model error representation</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Suffers from sampling error</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Table 1:** Important algorithmic differences. Green implies advantage and red implies disadvantage for a Gaussian system.
SEKF and DEnKF give similar average performance for single site in terms of RMSE and correlation coefficient (CC):

<table>
<thead>
<tr>
<th>Method</th>
<th>RMSE ($m^3/m^3$)</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open loop</td>
<td>$8.4 \times 10^{-3}$</td>
<td>0.90</td>
</tr>
<tr>
<td>SEKF</td>
<td>$7.0 \times 10^{-3}$</td>
<td>0.91</td>
</tr>
<tr>
<td>DEnKF</td>
<td>$6.9 \times 10^{-3}$</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 2: WG2 RMSE and CC for Sabres site.

- Additive inflation significantly improves DEnKF performance;
- Our perturbed precipitation does not improve DEnKF performance (not shown);
## DEnKF ensemble size - Sabres site

<table>
<thead>
<tr>
<th>Ensemble size</th>
<th>DEnKF WG2 RMSE ($m^3/m^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$8.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>6</td>
<td>$7.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>12</td>
<td>$6.9 \times 10^{-3}$</td>
</tr>
<tr>
<td>20</td>
<td>$7.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>50</td>
<td>$6.9 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

**Table 3:** DEnKF WG2 RMSE for various ensemble sizes.
Figure 3: WG2 open loop, analysis and observations ($m^3/m^3$).
Performance averages - 12 sites

- Same background/model error calibration used for all sites;
- Analysis performs better than open loop for 10 of 12 sites;

<table>
<thead>
<tr>
<th>Method</th>
<th>Average RMSE over 12 sites ($m^3/m^3$)</th>
<th>Average CC over 12 sites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open loop</td>
<td>$2.5 \times 10^{-2}$</td>
<td>0.81</td>
</tr>
<tr>
<td>SEKF</td>
<td>$2.0 \times 10^{-2}$</td>
<td>0.89</td>
</tr>
<tr>
<td>DEnKF</td>
<td>$2.0 \times 10^{-2}$</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 4: WG2 RMSE and CC averaged over 12 sites.
Figure 4: WG2 RMSE for DA methods ($m^3/m^3$).
Possible explanations for the results

- If the system were Gaussian (or quasi-Gaussian), DEnKF with the correct model-error specification should perform better than SEKF;
- Is the problem with our DEnKF related to our model error representation, or is it a non-Gaussian issue with the system?
- Probably a combination of both;
- Potential non-Gaussian issues:
  1. **Forcing (mainly precipitation)**: Widely known to exhibit highly non-Gaussian behaviour;
  2. **Bounded system** also non-Gaussian (drainage of water above field capacity, little water loss below wilting point);
  3. **Highly nonlinear WG1** variable;
- Potential model-error specification issues:
  1. **Over-simplified stochastic representation** of precipitation uncertainty (lognormal distribution);
  2. No uncertainty representation of **other forcing parameters**: radiative forcing, wind, etc...
  3. No uncertainty representation of **model parameters**.
Conclusions

- DEnKF and SEKF give similar performance for single ground-based observation;
- Model and observation agreement unusually good for single site → analysis corrections small;
- Results show similar performance of the two methods averaged over 12 sites;
- Large changes in soil moisture content between the seasons could explain why WG2 analysis errors are twice as large in spring and autumn than in winter;
- DEnKF unable to improve on SEKF due to deficient model-error representation and/or non-Gaussian issues with model.
Future work

- Investigate issues caused by the limitations in the linear approximations made by the DA methods - See Alina’s poster;
- Comparison between the two methods for different temporal observation frequencies (every 6 hours - every 3 days);
- Improve understanding and representation of model error;
- Investigate horizontal correlations with a 2D grid;
- Increase the vertical resolution of the model using a diffusive hydrological scheme.


