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Generation of artificial inflow turbulence including scalar fluctuation for LES based on Cholesky decomposition

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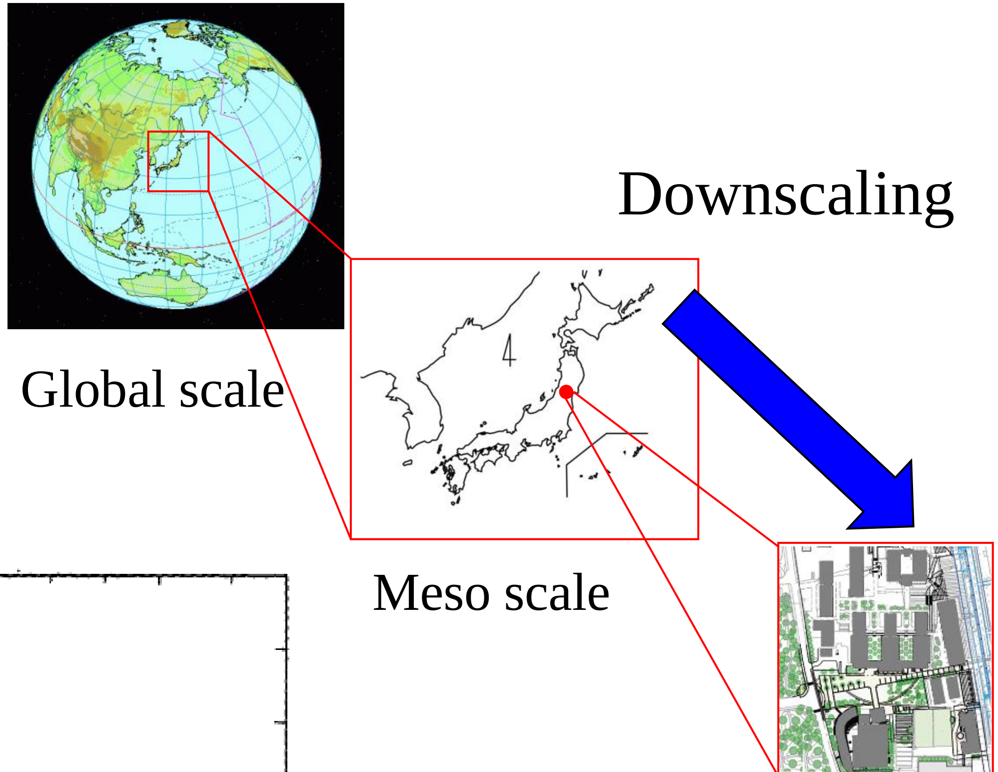
Issue regarding the coupling of LES with MMM

In large-eddy simulation cases, an inflow turbulence which satisfies not only the turbulent statistics but also the instantaneous turbulent fluctuation should be generated.



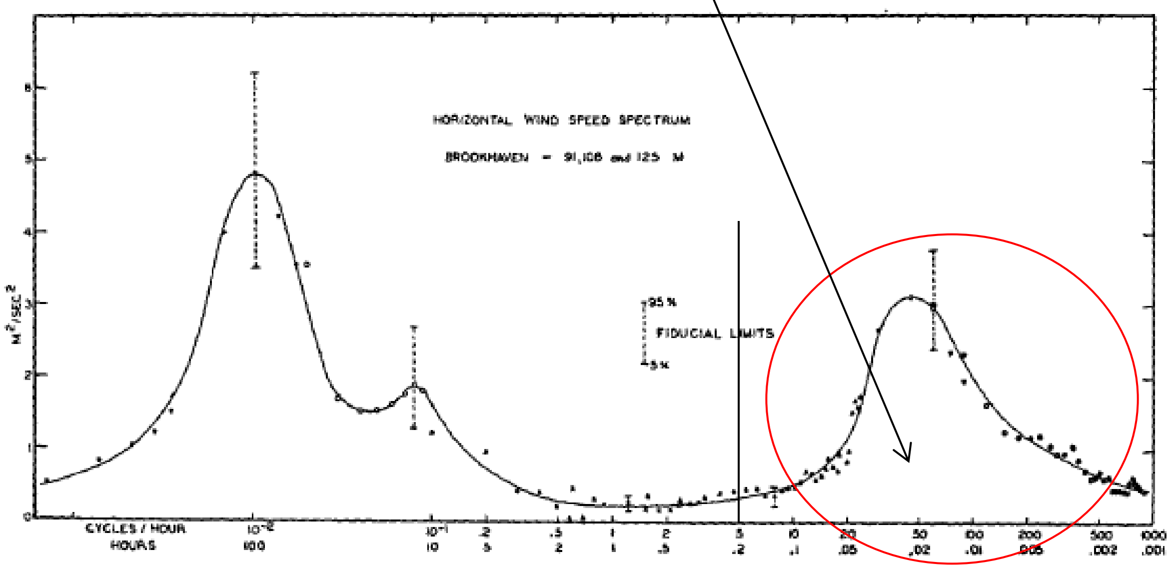


Fig. 1. Horizontal wind-speed spectrum at Brookhaven National Laboratory at about 100-m height. (See table 1 for date and time.)

Van der Hoven - Journal of Meteorology, 1957

Recently, increased access to computing power has led to several attempts to couple large-eddy simulation (LES) and MMM.

Approaches to generating inflow turbulence

The approaches to generating inflow turbulence can be divided into two types:

1) Storing the time history of velocity fluctuations obtained from a preliminary or recycling LES computation

   Lund et al., 1998; Kataoka and Mizuno, 2002

2) Artificially generating inflow turbulence which prescribes turbulent statistics without conducting LES computations

   Lee et al., 1992; Iizuka et al., 1999;

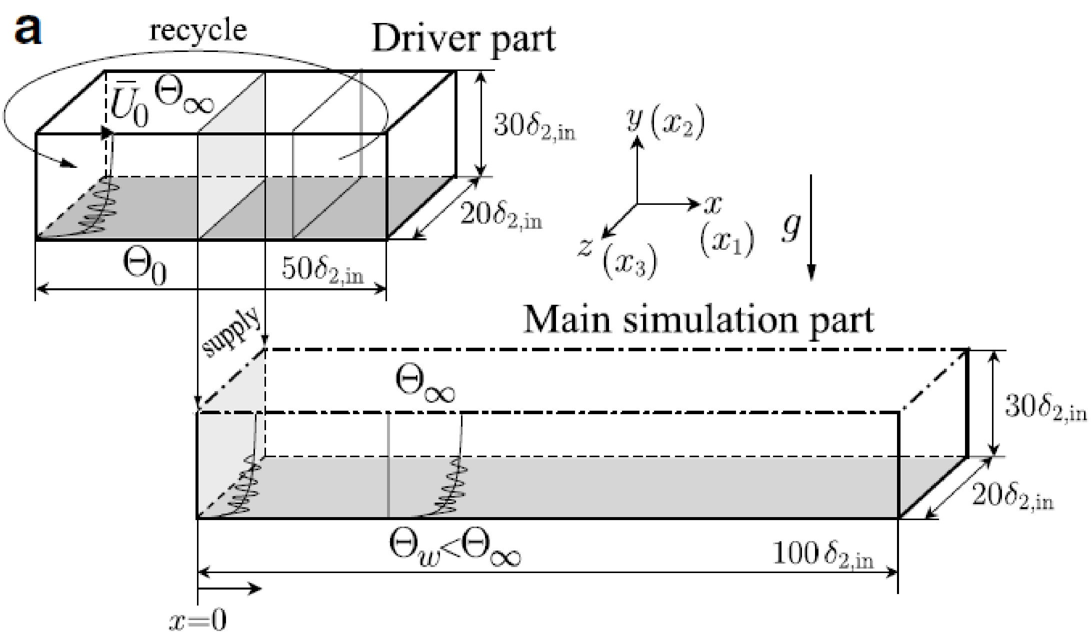
   Klein et al., 2003; Xie and Castro, 2008; Kondo and Iizuka, 2012

   Jarrin et al. ,2006 etc.

Non-isothermal LES

In recent years, non-isothermal LES computations within boundary layers have been carried.

When LES is applied to a non-isothermal field, not only the inflow velocity fluctuation but also the temperature fluctuation should be reproduced.



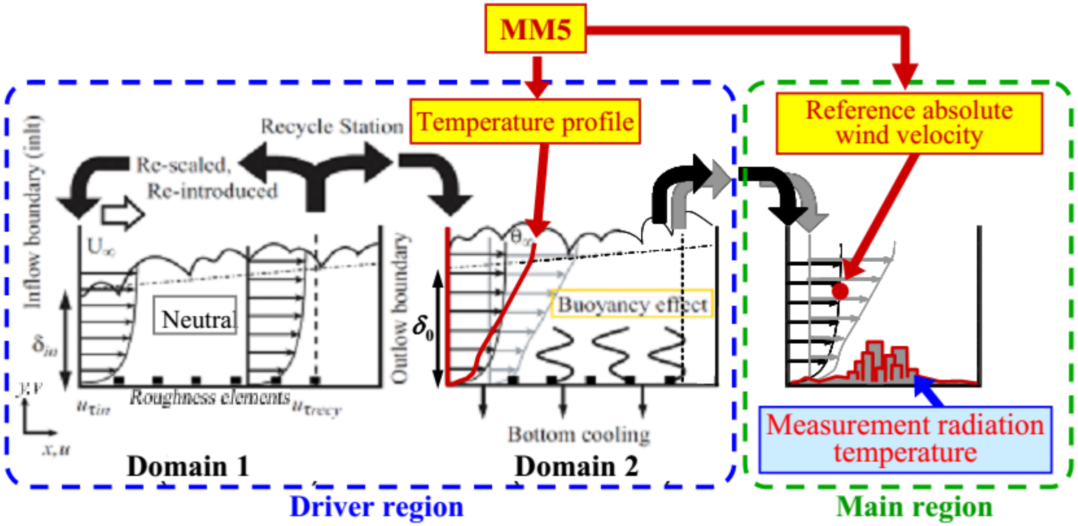
The temperature was treated as passive scalar in the driver section.

Kong, H., Choi, H., & Lee, J. S. (2000) *Physics of Fluids*, *12*(10)

Hattori, H., Houra, T., & Nagano, Y. (2007). *International Journal of Heat and Fluid Flow*, *28*(6), 1262–1271.

Non-isothermal LES

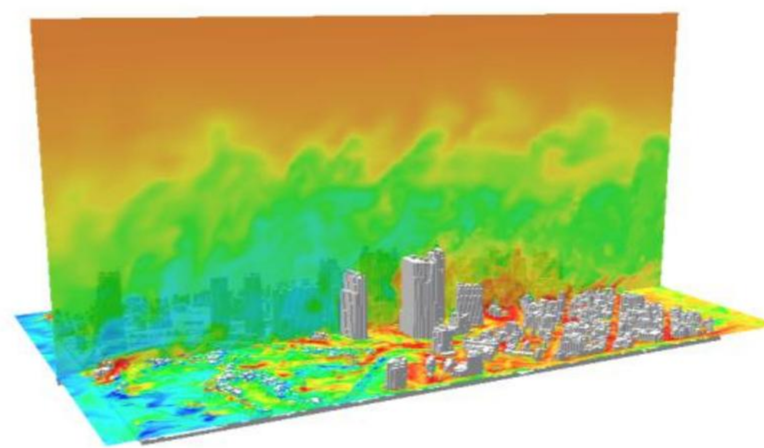
In building scale LES cases,



Tamura et al. (2012) conducted a non-isothermal LES in Tokyo area with a generated temperature fluctuation.

Few study have been conducted.

In addition, the generation method for temperature fluctuation based on preliminary LES sometimes consumes much computational cost.



Tamura, T., Nozu, T., Okuda, Y., Ohashi, M., Umakawa, H.: The USB proceedings of 8th International Conference on Urban Climates, Aug. 2012.

Objective of this study

This paper proposes

a new method of artificially generating turbulent fluctuations
in wind velocity and scalar quantities
such as temperature and contaminant
based on the Cholesky decomposition of the time-averaged turbulent flux tensors of momentum and scalar.

The method was validated by applying it to LES computations of contaminant dispersion in a half-channel flow.

1. Background and objective

2. New method of generating inflow turbulence including scalar fluctuation

3. Outline of LES computations

4. Conclusions

Definition of values

In this study,

we express the values of wind velocity and scalar as f_i,

the time-averaged values of f_i as $<f_i>$,

and the deviation from the time-averaged value as f_i':

$$f_i = \langle f_i \rangle + f_i'$$

i = 1, 2, 3: the wind velocity components

in the streamwise, lateral, and vertical directions (u, v, w)

i = 4: the scalar value ϕ.

Matrix of the turbulent fluxes of momentum and scalar

A regular matrix of the turbulent fluxes of momentum and scalar, R_{ij}, is defined as

$$R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle & \langle u'\phi' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle & \langle v'\phi' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle & \langle w'\phi' \rangle \\ \langle \phi'u' \rangle & \langle \phi'v' \rangle & \langle \phi'w' \rangle & \langle \phi'\phi' \rangle \end{pmatrix}$$

i = 1, 2, 3: the wind velocity components

in the streamwise, lateral, and vertical directions (u, v, w)

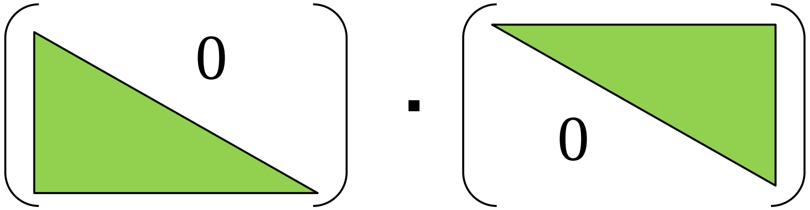
i = 4 indicates the scalar value ϕ.

Cholesky decomposition of R_{ij}

$$R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle & \langle u'\phi' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle & \langle v'\phi' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle & \langle w'\phi' \rangle \\ \langle \phi'u' \rangle & \langle \phi'v' \rangle & \langle \phi'w' \rangle & \langle \phi'\phi' \rangle \end{pmatrix}$$

Cholesky decomposition

$$R_{ik} = a_{ik} \cdot a_{kj} = a \cdot a^{\mathrm{T}} =$$



A lower triangular matrix, a_{ij}, is obtained.

$$a_{ij} = \begin{pmatrix} \sqrt{R_{11}} & 0 & 0 & 0 \\ R_{21}/a_{11} & \sqrt{R_{22} - a_{21}^2} & 0 & 0 \\ R_{31}/a_{11} & \left(R_{32} - a_{21}a_{31}\right)/a_{22} & \sqrt{R_{33} - a_{31}^2 - a_{32}^2} & 0 \\ R_{41}/a_{11} & \left(R_{42} - a_{21}a_{41}\right)/a_{22} & \left(R_{43} - a_{31}a_{41} - a_{32}a_{42}\right)/a_{33} & \sqrt{R_{44} - a_{41}^2 - a_{42}^2 - a_{43}^2} \end{pmatrix}$$

Expression of fluctuations using a_{ij}

With the lower triangular matrix, a_{ij} and a variable Ψ_j satisfying

$\langle \Psi_j \rangle = 0$ and $\langle \Psi_i \Psi_j \rangle = \delta_{ij}$,

the fluctuations, f'_i, can be rewritten as

$$f_i = \langle f_i \rangle + \underline{f'_i} = \langle f_i \rangle + \underline{a_{ij} \Psi_j}$$

The transformation was originally proposed by Lund et al. (1998) using Reynolds stress tensor.

$$R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle \end{pmatrix}$$

Lund T. S., Wu X., Squires K. D., 1998: Generation of turbulent inflow data for spatially-developing boundary layer simulations, *Journal of Computational Physics*, 140(2), 233-258

Extension of the transformation

We extended the transformation to consider the turbulent fluxes of scalar.

$$R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle \end{pmatrix} \Rightarrow R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle & \langle u'\phi' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle & \langle v'\phi' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle & \langle w'\phi' \rangle \\ \langle \phi'u' \rangle & \langle \phi'v' \rangle & \langle \phi'w' \rangle & \langle \phi'\phi' \rangle \end{pmatrix}$$

$$a_{ij} = \begin{pmatrix} \sqrt{R_{11}} & 0 & 0 & 0 \\ R_{21}/a_{11} & \sqrt{R_{22} - a_{21}^2} & 0 & 0 \\ R_{31}/a_{11} & \left(R_{32} - a_{21}a_{31}\right)/a_{22} & \sqrt{R_{33} - a_{31}^2 - a_{32}^2} & 0 \\ R_{41}/a_{11} & \left(R_{42} - a_{21}a_{41}\right)/a_{22} & \left(R_{43} - a_{31}a_{41} - a_{32}a_{42}\right)/a_{33} & \sqrt{R_{44} - a_{41}^2 - a_{42}^2 - a_{43}^2} \end{pmatrix}$$

How to give the value of Ψ_j

$$f_i = \langle f_i \rangle + f_i' = \langle f_i \rangle + a_{ij}\Psi_j$$

$$\begin{cases} \langle \Psi_j \rangle = 0 \\ \langle \Psi_i \Psi_j \rangle = \delta_{ij} \end{cases}$$

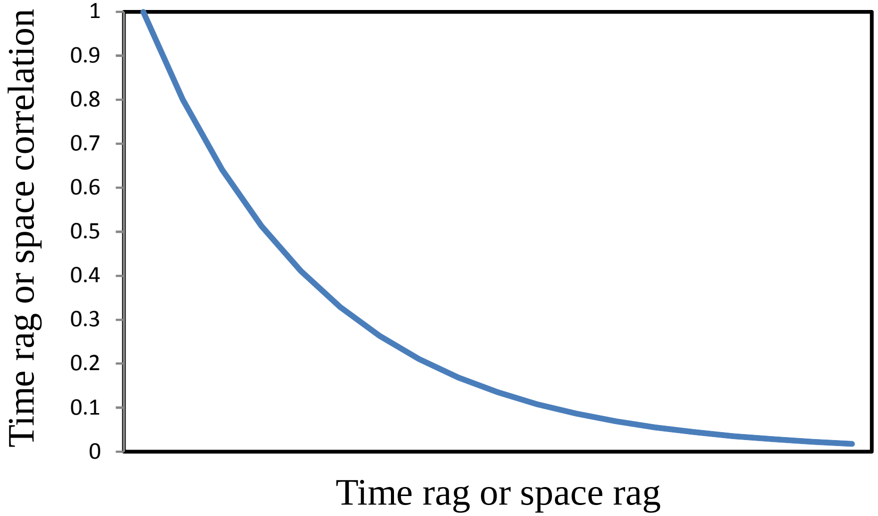
To impose time and space correlations for each component of the fluctuations, the two-dimensional digital-filter method proposed by Xie and Castro (2008) and then revised by Kondo and Iizuka (2012) was employed.

Prescribed time and space correlations

The prescribed time and space correlations are assumed using exponential functions with an integral time scale, *T*, and a length scale, *L*:

$$\frac{\langle \Psi_j(t)\Psi_j(t+\tau)\rangle}{\langle \Psi_j(t)\Psi_j(t)\rangle} = \exp\left(-\frac{\tau}{T}\right)$$

$$\frac{\langle \Psi_j(r)\Psi_j(r+\lambda)\rangle}{\langle \Psi_j(r)\Psi_j(r)\rangle} = \exp\left(-\frac{\lambda}{L}\right)$$



τ: Time rag

λ: space rag

The time advances of generated fluctuations

The time advances of artificially generated fluctuations

on a grid point (m, n) are expressed as

$$\Psi_j(t+\Delta t, m, n) = \Psi_j(t, m, n)\exp\left(-\frac{\Delta t}{T}\right) + \psi_j(t+\Delta t, m, n)\left\{1-\exp\left(-\frac{2\Delta t}{T}\right)\right\}^{1/2}$$

$$\psi_j(t, m, n) = \sum_{m'=1}^{N_y}\sum_{n'=1}^{N_z} b_{m'} b_{n'} r_{m+m', n+n'}$$

r: a random number satisfying $<r_j> = 0$ and $<r_i\, r_j> = \delta_{ij}$
N_y, N_z : number of grid points included
in the generated plane in each direction
b_k : a digital-filter coefficient for the integral length scale
in the generated plane in each direction

The time advances of generated fluctuations

$$\Psi_j(t+\Delta t, m, n) = \Psi_j(t, m, n)\exp\left(-\frac{\Delta t}{T}\right) + \psi_j(t+\Delta t, m, n)\left\{1-\exp\left(-\frac{2\Delta t}{T}\right)\right\}^{1/2}$$

$$\psi_j(t, m, n) = \sum_{m'=1}^{N_y}\sum_{n'=1}^{N_z} b_{m'} b_{n'} r_{m+m', n+n'}$$

b_k : a digital-filter coefficient for the integral length scale in the generated plane in each direction

According to the method proposed by Xie and Castro (2008), two-dimensional random data are filtered to generate a set of two-dimensional data with the prescribed spatial correlation.

Then, these data are combined with those from the previous time step by using two weighting factors based on the exponential functions.

Procedure of generation of fluctuations

By substituting ψ_j as obtained using a new dataset of random numbers

$$\psi_j(t,m,n) = \sum_{m'=1}^{N_y}\sum_{n'=1}^{N_z} b_{m'} b_{n'} r_{m+m',n+n'}$$

into the equation below for each time step,

$$\Psi_j(t+\Delta t,m,n) = \Psi_j(t,m,n)\exp\left(-\frac{\Delta t}{T}\right) + \psi_j(t+\Delta t,m,n)\left\{1-\exp\left(-\frac{2\Delta t}{T}\right)\right\}^{1/2}$$

Ψ_j for the next time step is obtained. Then, the fluctuations, f_i, are given by substituting Ψ_j into

$$f_i = \langle f_i \rangle + f_i' = \langle f_i \rangle + a_{ij}\Psi_j$$

Storing Ψ_j temporarily.

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2. New method of generating inflow turbulence including scalar fluctuation

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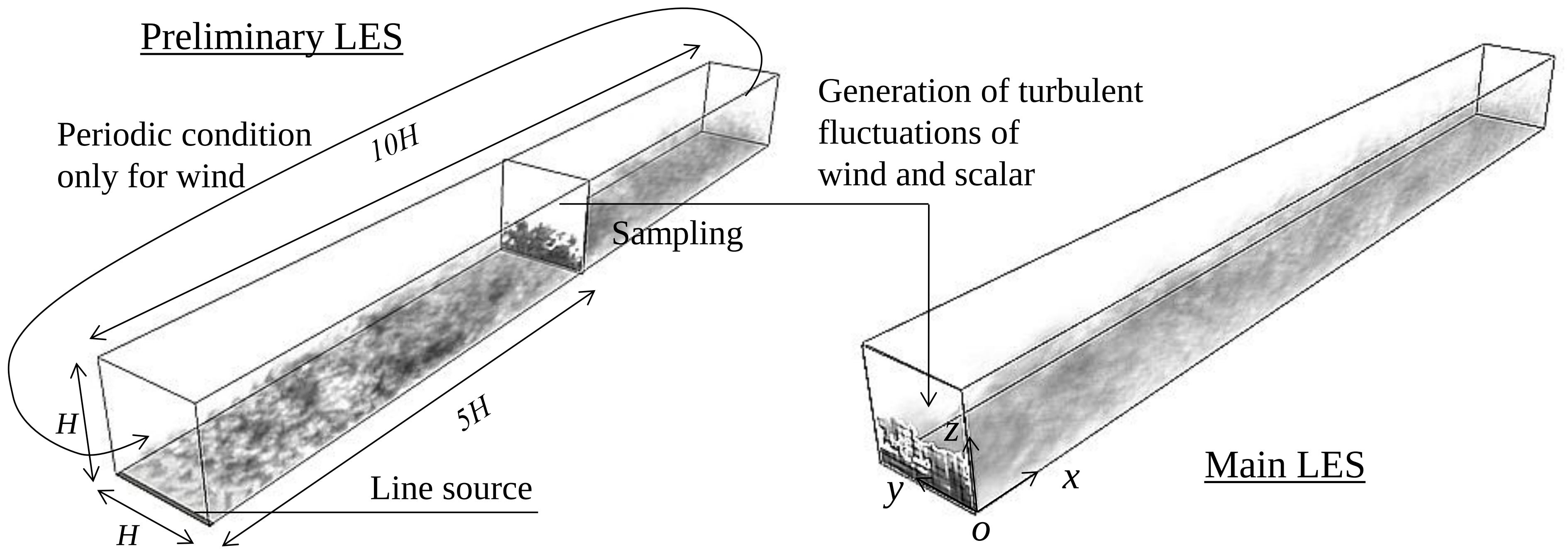
A priori LES computations

A priori LES computations for a half-channel were carried out to validate the reproducibility of the flow and dispersion fields by applying the artificially generated wind and scalar fluctuations as an inflow boundary condition.

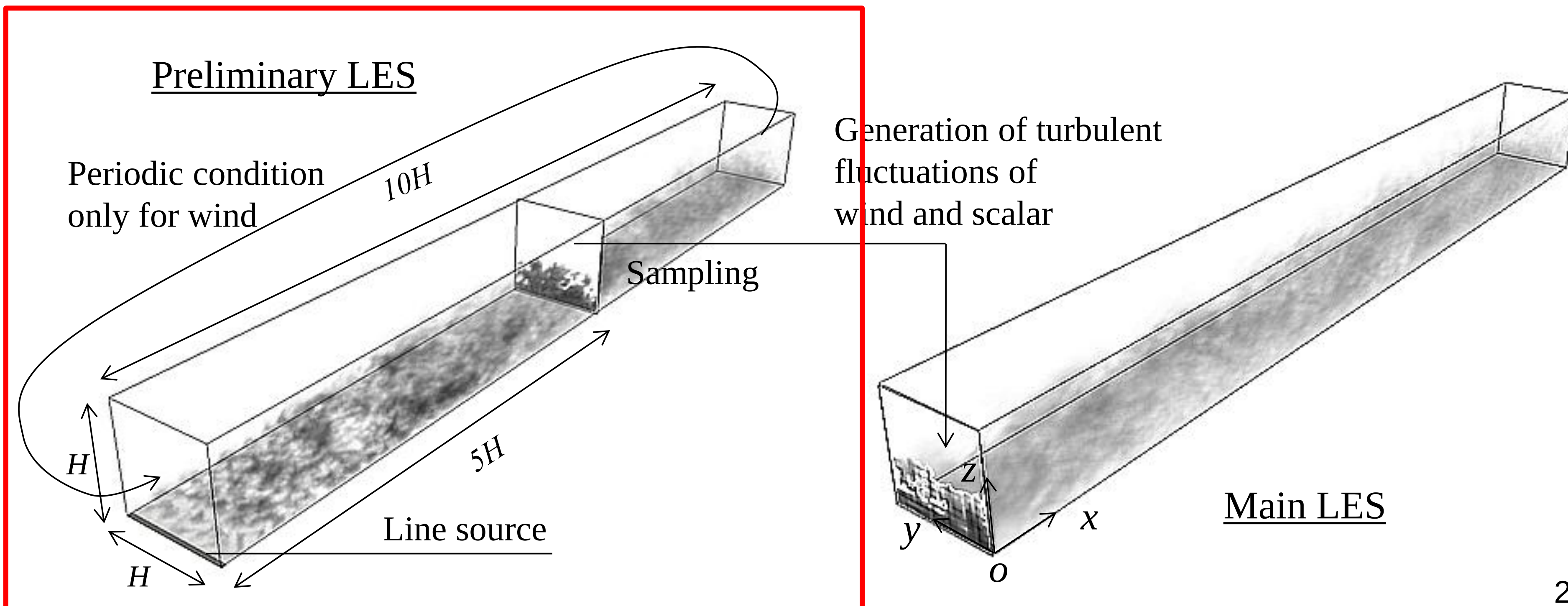


Outline of **preliminary** LES computation

First, a preliminary LES computation was conducted to obtain the turbulent statistics.

A line source was placed on the ground cell immediately behind the inflow boundary and a passive scalar was emitted.

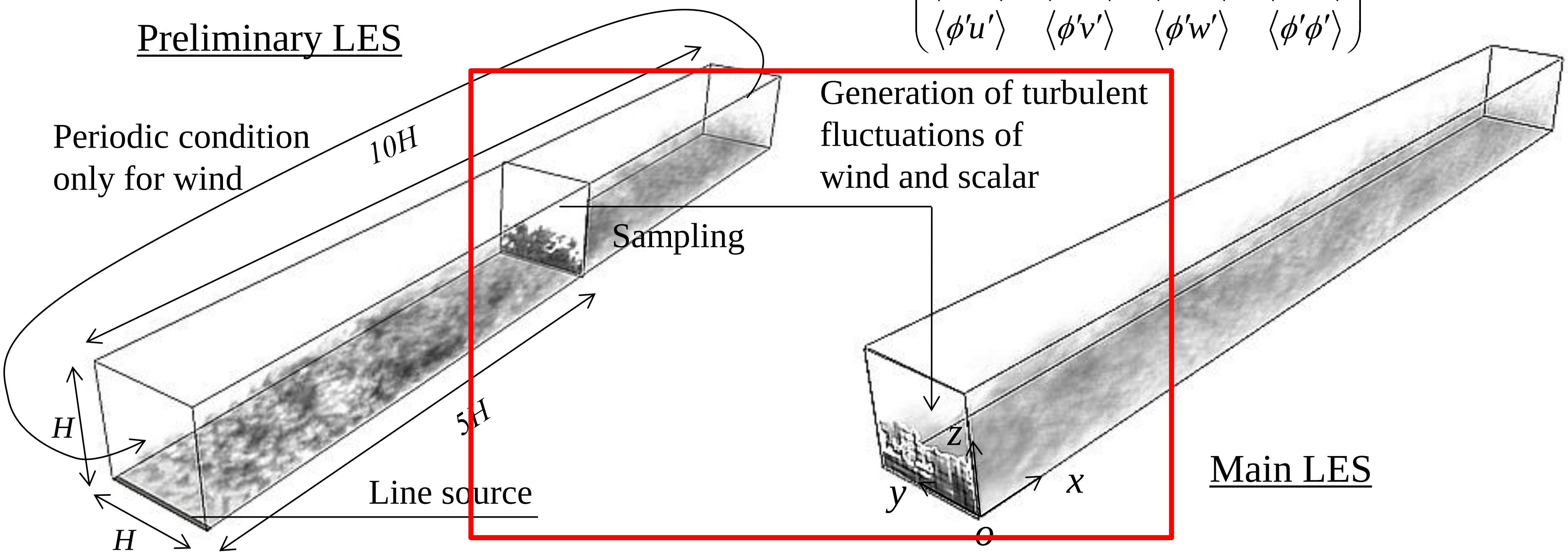
The time series of the turbulent fluctuations of the wind and scalar values were stored on the *y*-*z* plane at $x = 5.0H$.



Artificially generation of fluctuations

Then, the fluctuations of the wind and scalar values were artificially generated based on the Cholesky decomposition of the time-averaged turbulent flux tensor of momentum and scalar which were obtained from the database collected at x = 5.0H in the preliminary simulation.

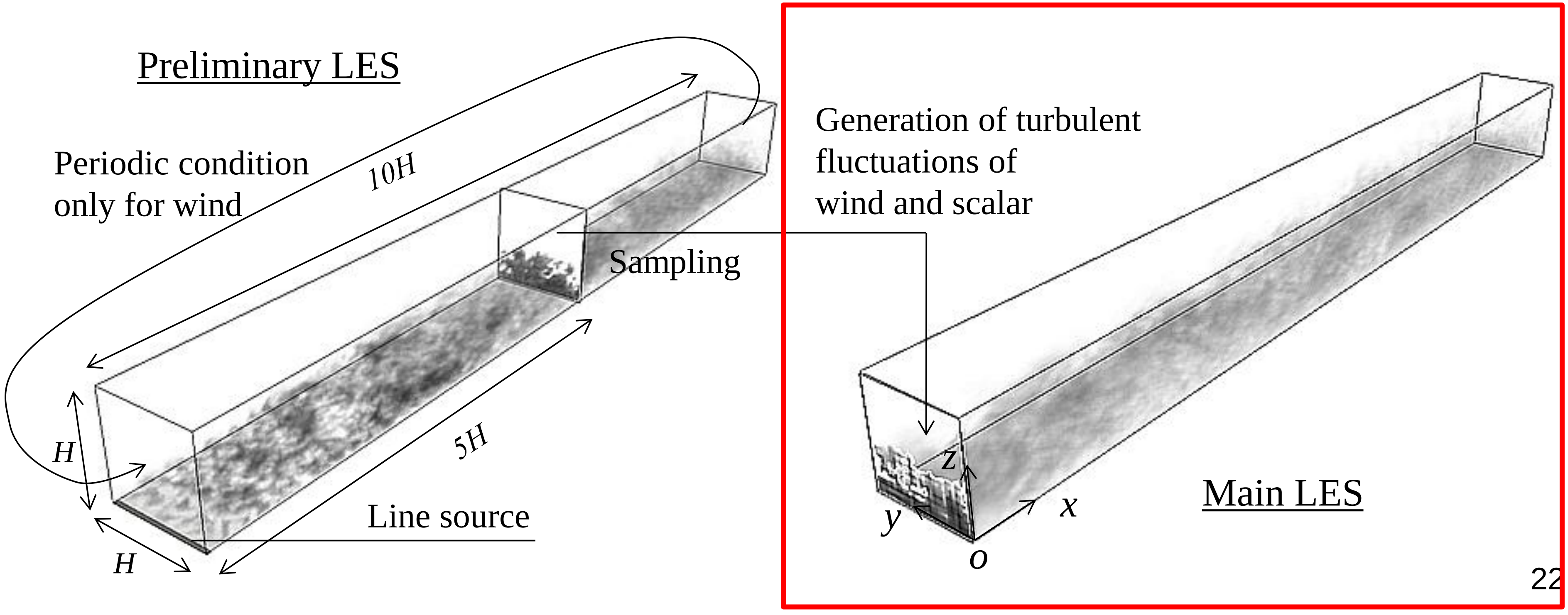
$$R_{ij} = \begin{pmatrix} \langle u'u' \rangle & \langle u'v' \rangle & \langle u'w' \rangle & \langle u'\phi' \rangle \\ \langle v'u' \rangle & \langle v'v' \rangle & \langle v'w' \rangle & \langle v'\phi' \rangle \\ \langle w'u' \rangle & \langle w'v' \rangle & \langle w'w' \rangle & \langle w'\phi' \rangle \\ \langle \phi'u' \rangle & \langle \phi'v' \rangle & \langle \phi'w' \rangle & \langle \phi'\phi' \rangle \end{pmatrix}$$



Outline of **main** LES computation

Finally, the main LES computation was carried out with the artificially generated turbulent fluctuations as the inflow boundary condition of the main computation.

The reproducibility of the flow and dispersion fields when applying the artificially generated wind and scalar fluctuations as inflow boundary conditions was validated.



Integral length and time scales used in this study

$$\frac{\langle \Psi_j(t)\Psi_j(t+\tau)\rangle}{\langle \Psi_j(t)\Psi_j(t)\rangle} = \exp\left(-\frac{\tau}{T}\right) \qquad \frac{\langle \Psi_j(r)\Psi_j(r+\lambda)\rangle}{\langle \Psi_j(r)\Psi_j(r)\rangle} = \exp\left(-\frac{\lambda}{L}\right)$$

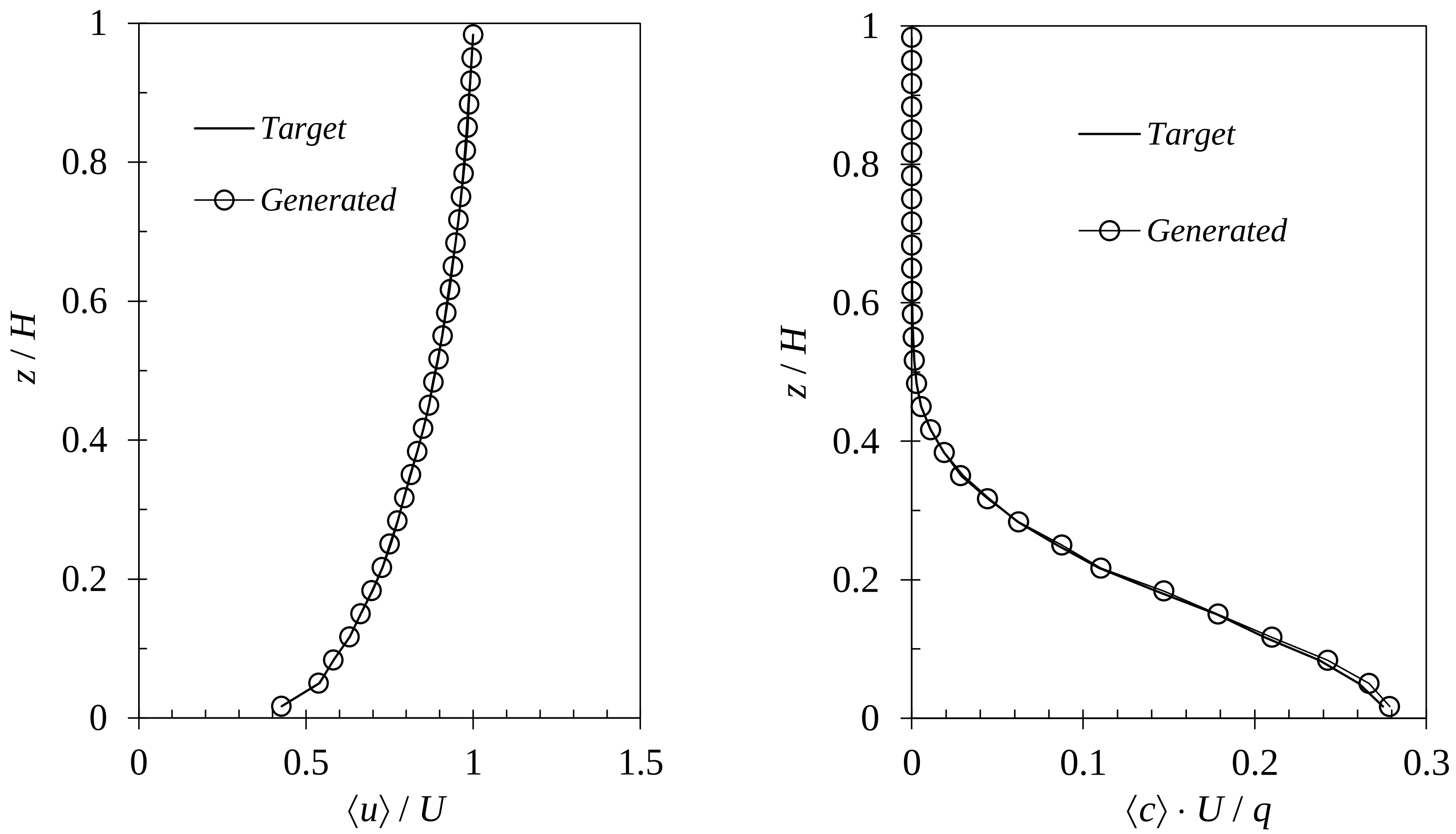
The integral length scales for prescribing the space correlations of the turbulent fluctuations of wind velocity
were assumed to be $L = 0.15H$ (H: Domain height).

The integral time scales for prescribing the time correlations of the turbulent fluctuations were given based on the frozen turbulence approximation known as "Taylor's hypothesis":

$$T = L/U$$

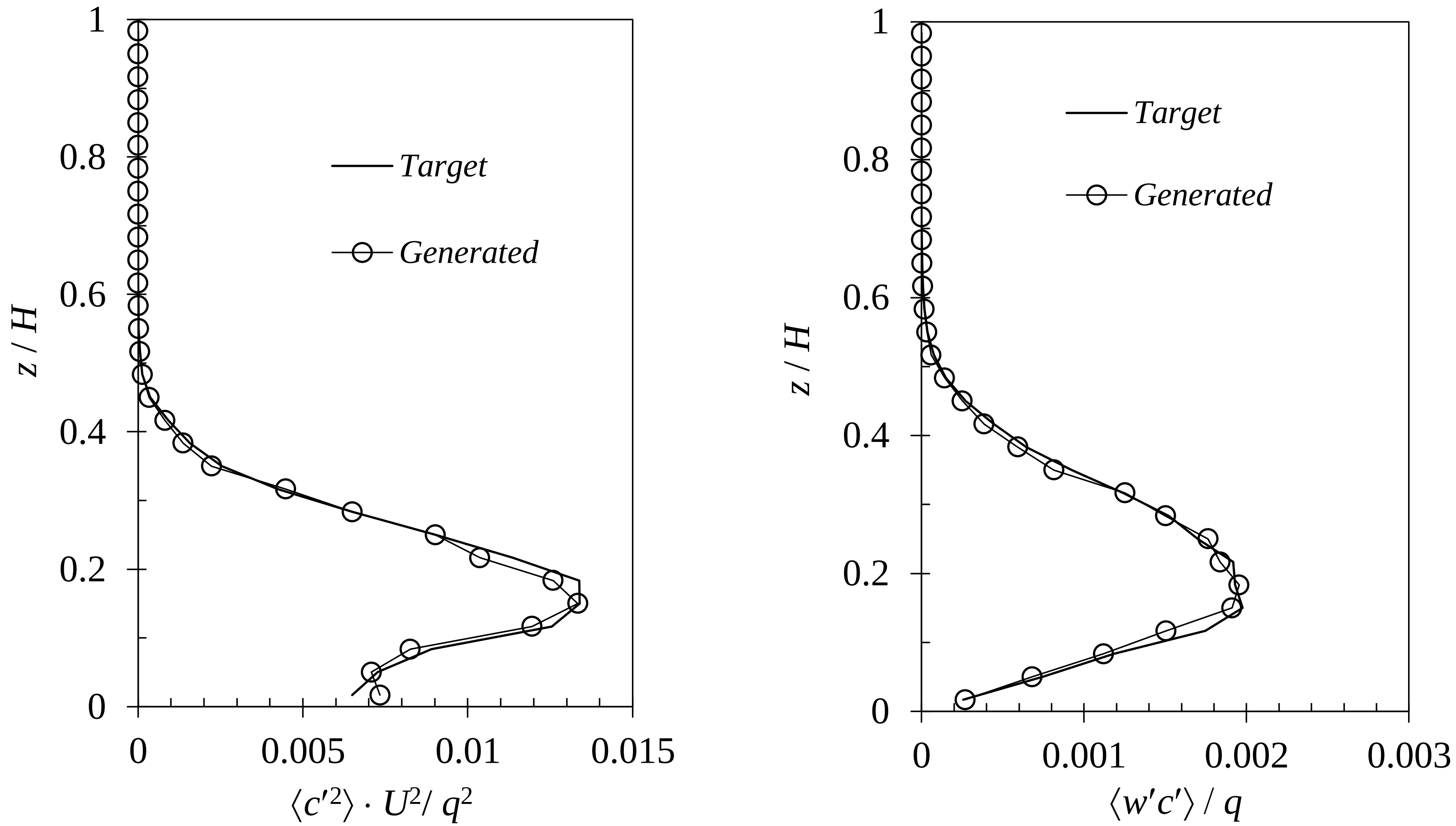
It is assumed that the integral length and time scales for scalar dispersion are equal to those for the wind velocity.

Turbulent statistics for generated mean values



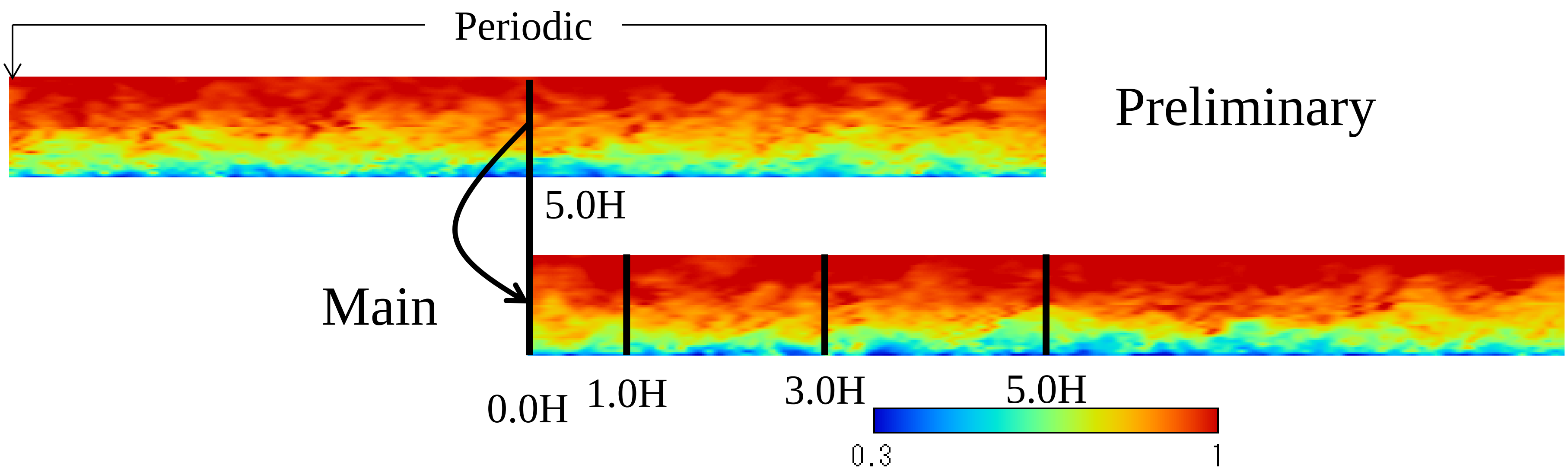
The generated mean wind velocity and concentration are in agreement completely with the targeted values.

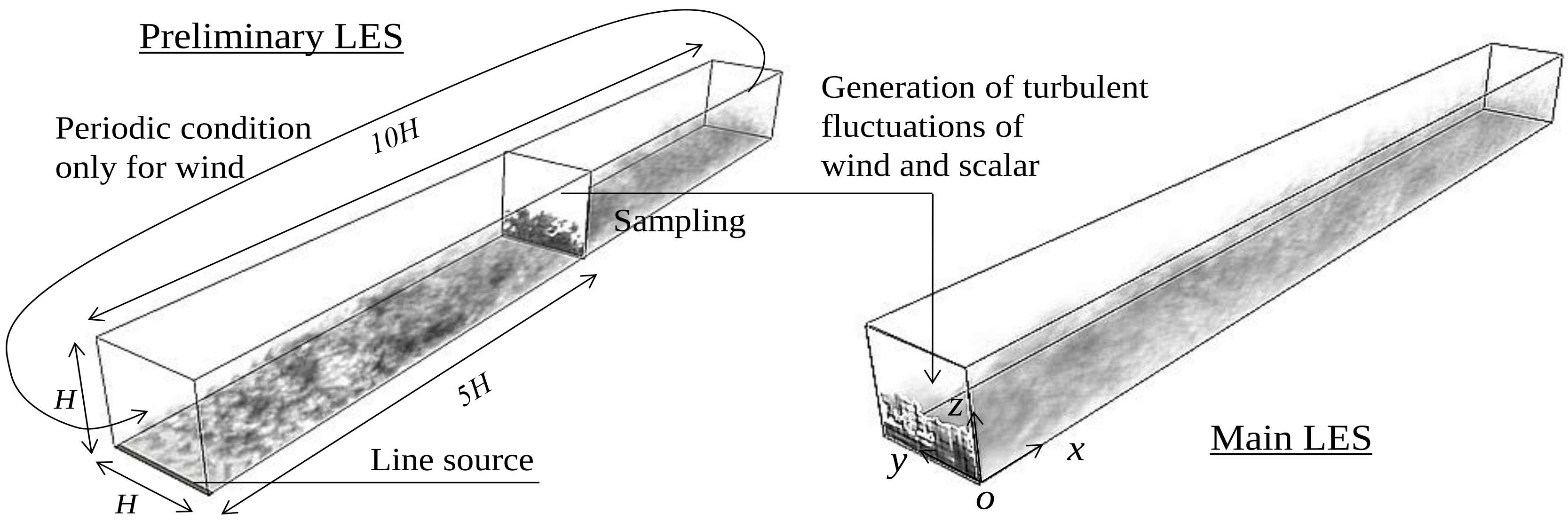
Turbulent statistics for generated fluctuations



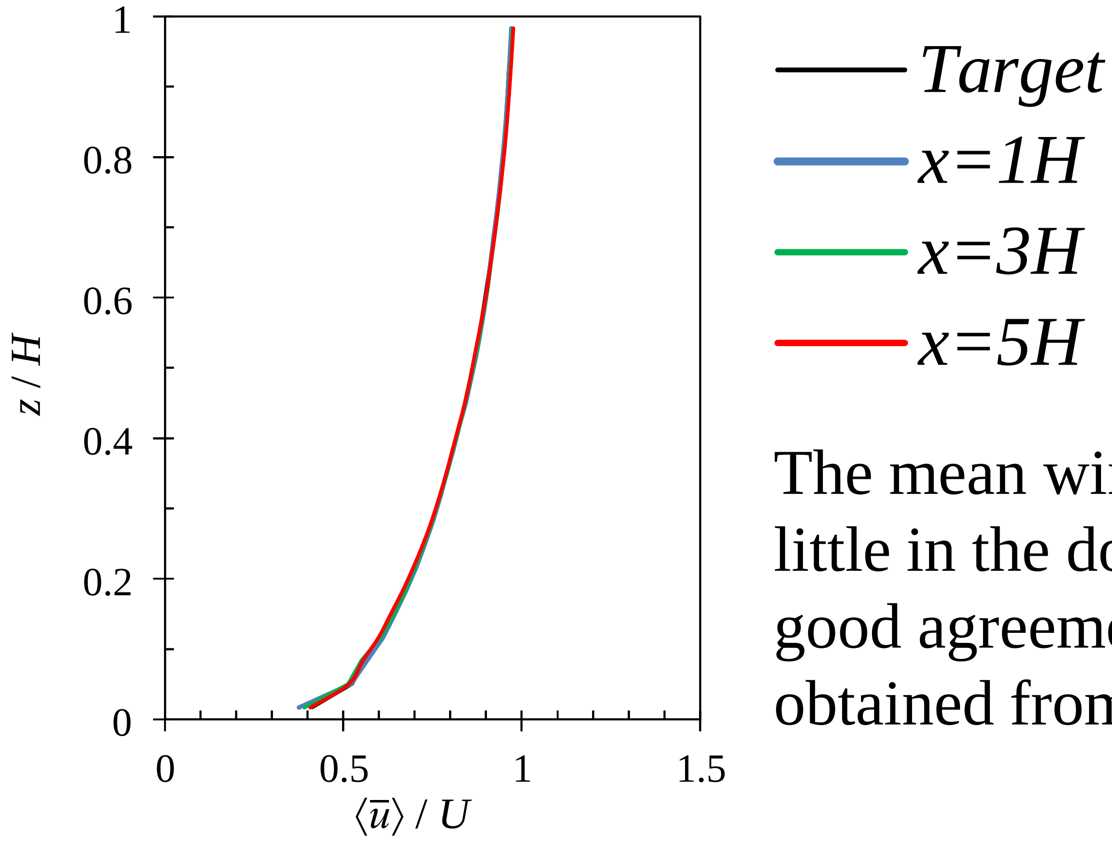
The variance of the generated concentration and the turbulent scalar flux in the vertical direction as obtained from the artificial generation are also in good agreement with the targeted values.

Comparison of flow field

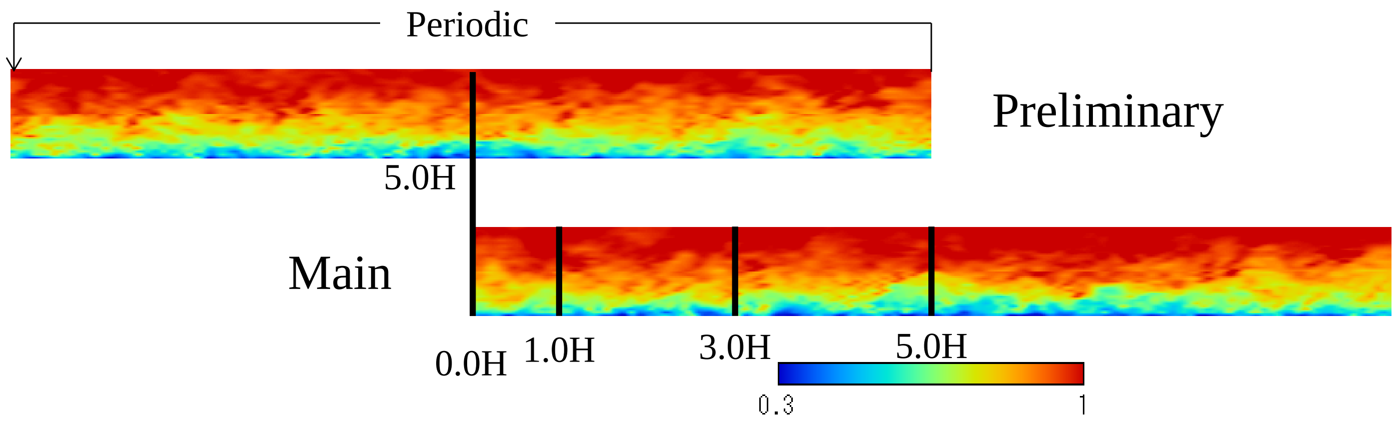




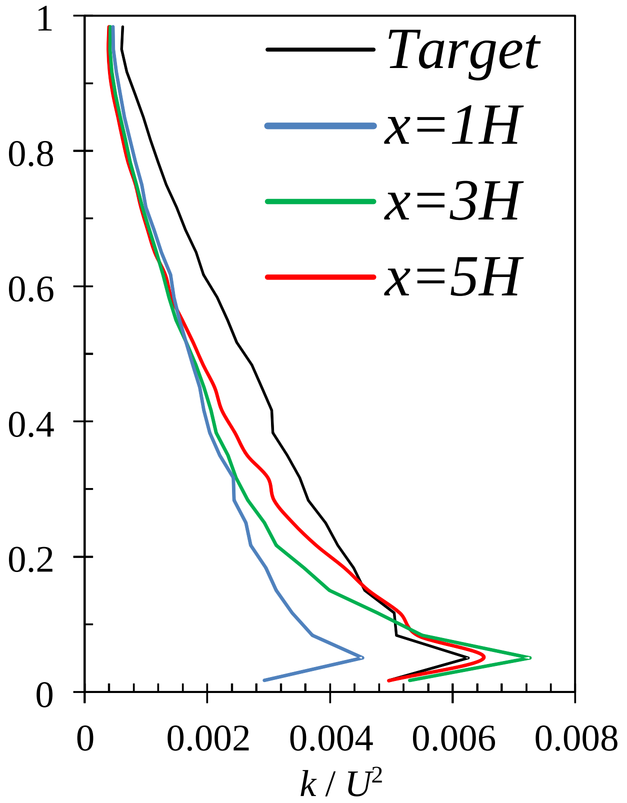
Streamwise change of mean wind velocity



The mean wind velocity changes very little in the downstream region and is in good agreement with the target value obtained from the preliminary simulation.

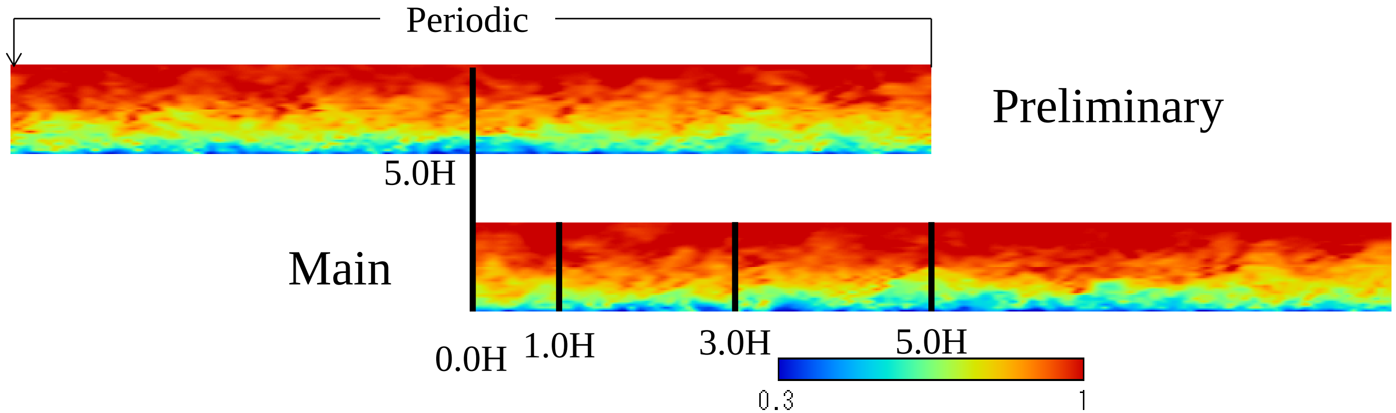


Streamwise change of TKE in grid scale

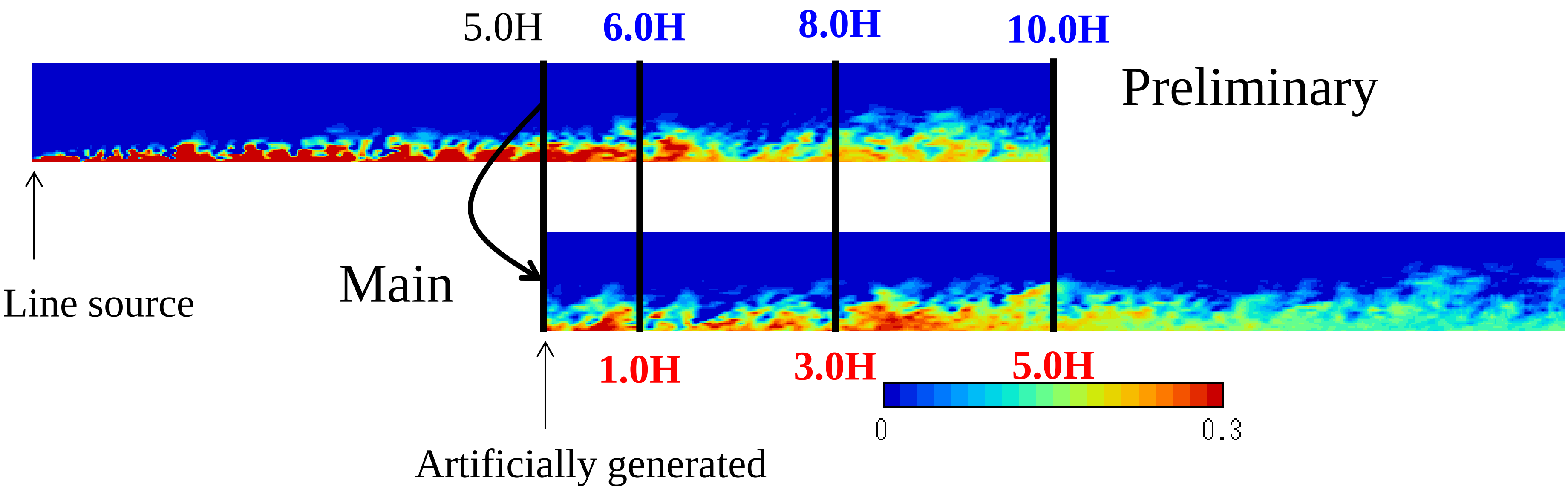


The turbulent kinetic energy in the grid scale at x = 1.0H is rapidly damped by 40%, relative to the target value.

The causes are suspected to be related to the artificially generated fluctuations not generally being able to satisfy the continuity and momentum equations (Xie and Castro, 2008; Kondo and Iizuka, 2012).

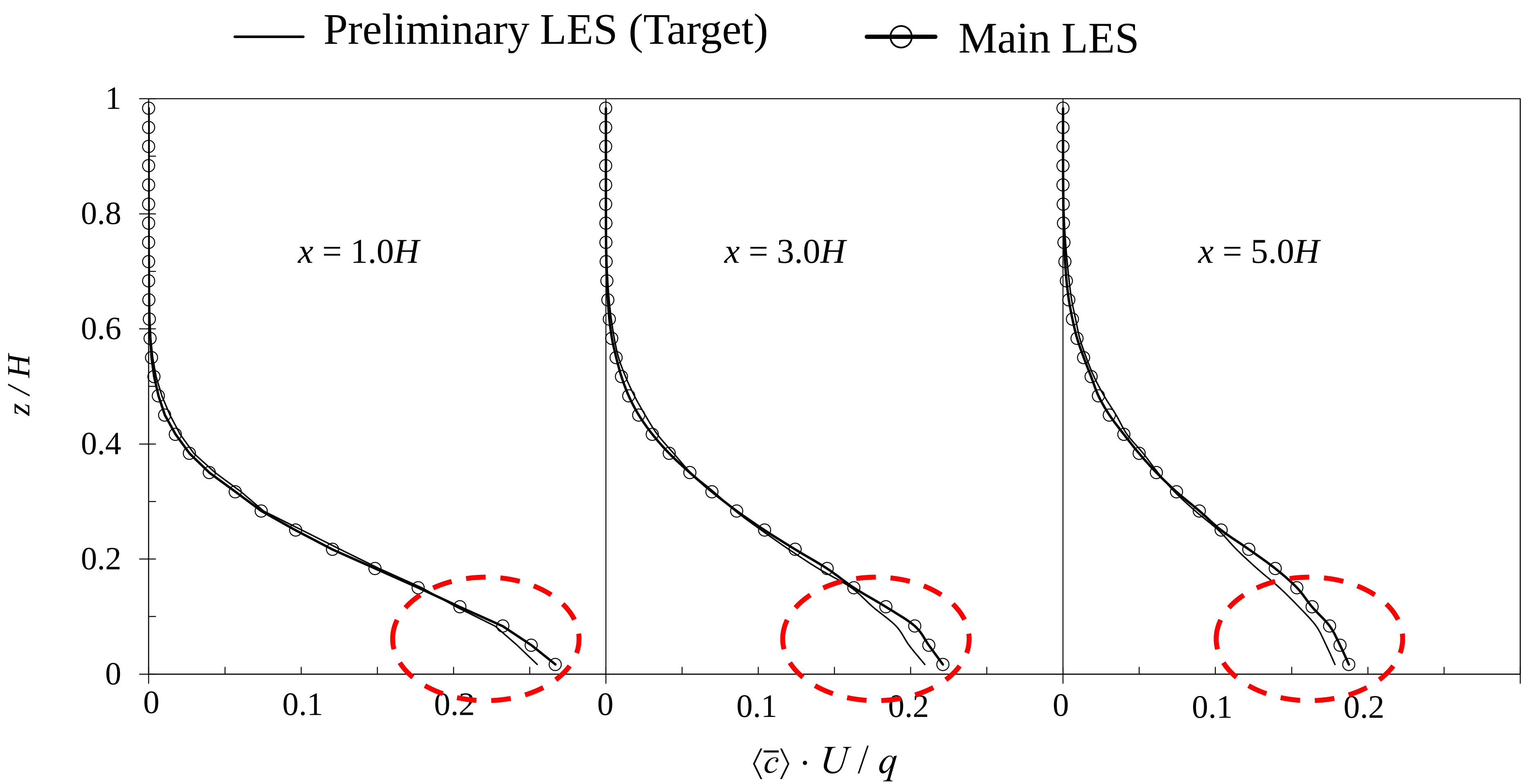


Comparison of concentration field



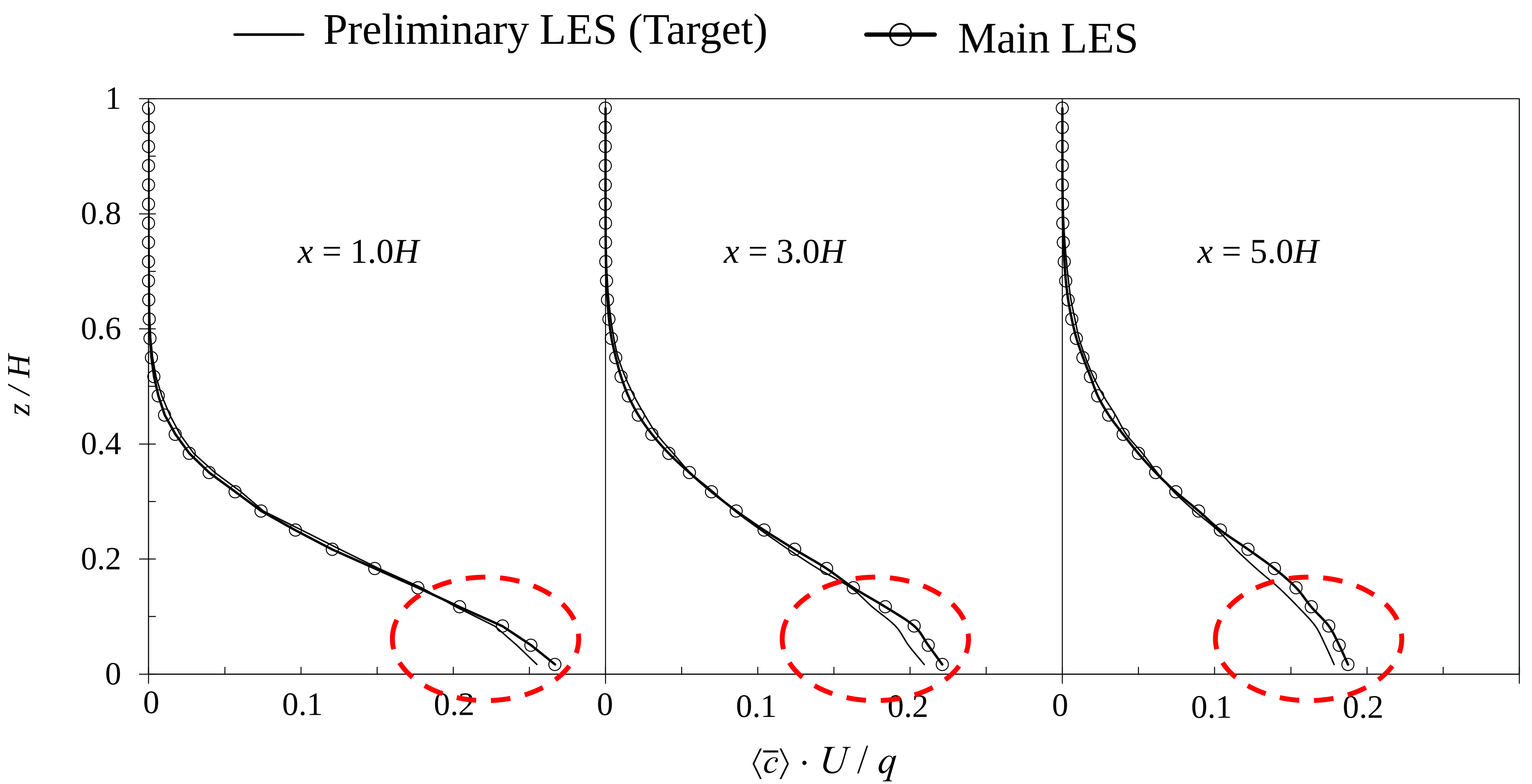
By comparing the results for the concentration field **at x = 1.0H, 3.0H, and 5.0H in the main calculation** with that obtained with **x = 6.0H, 8.0H, and 10.0H in the preliminary calculation**, the reproducibility of the flow and dispersion fields when applying the artificially generated wind and scalar fluctuations as inflow boundary conditions was validated.

Streamwise change of mean concentration



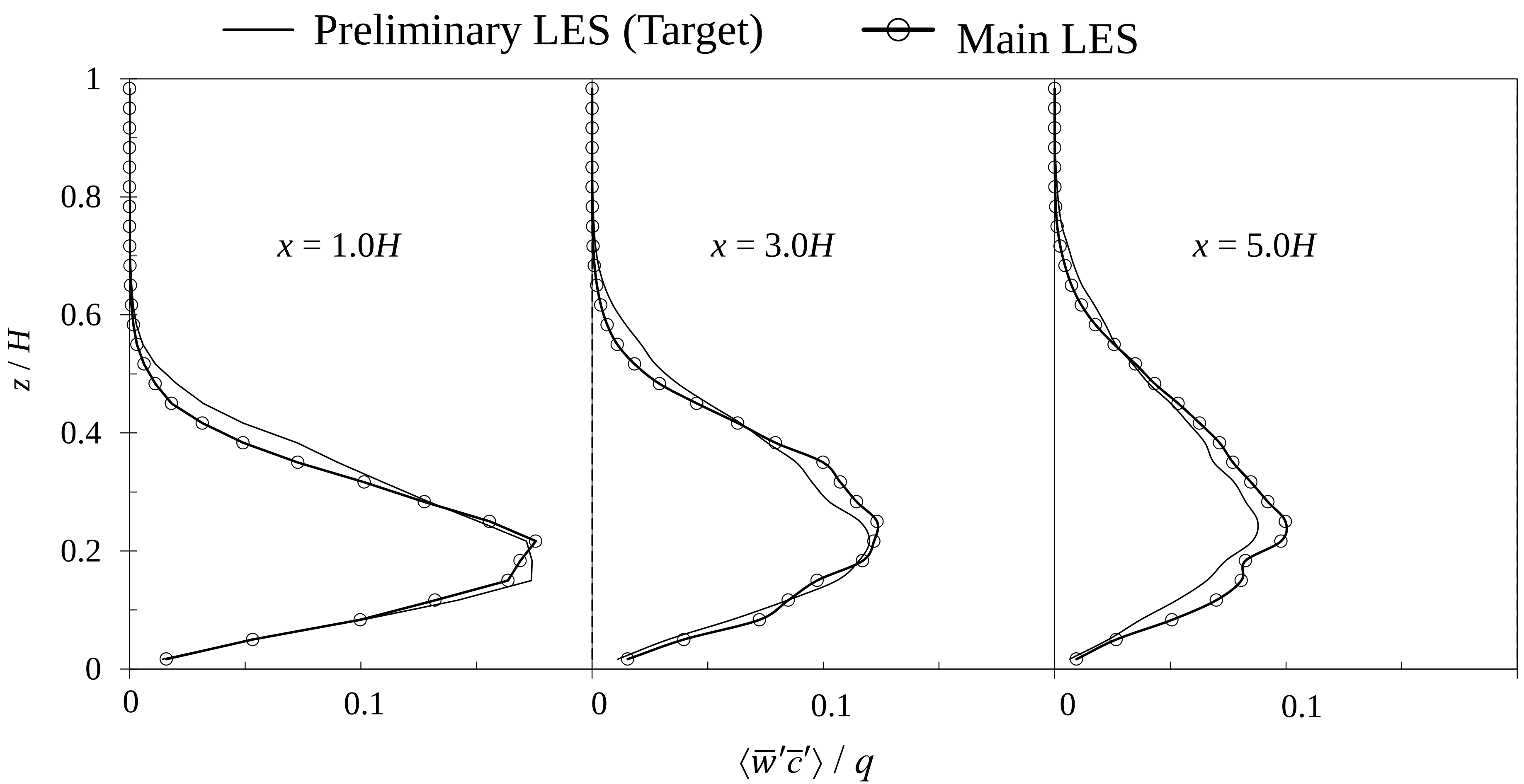
The result of the mean concentration obtained from the main simulation with artificially generated fluctuations is slightly larger than the result of the preliminary simulation near the surface.

Streamwise change of mean concentration



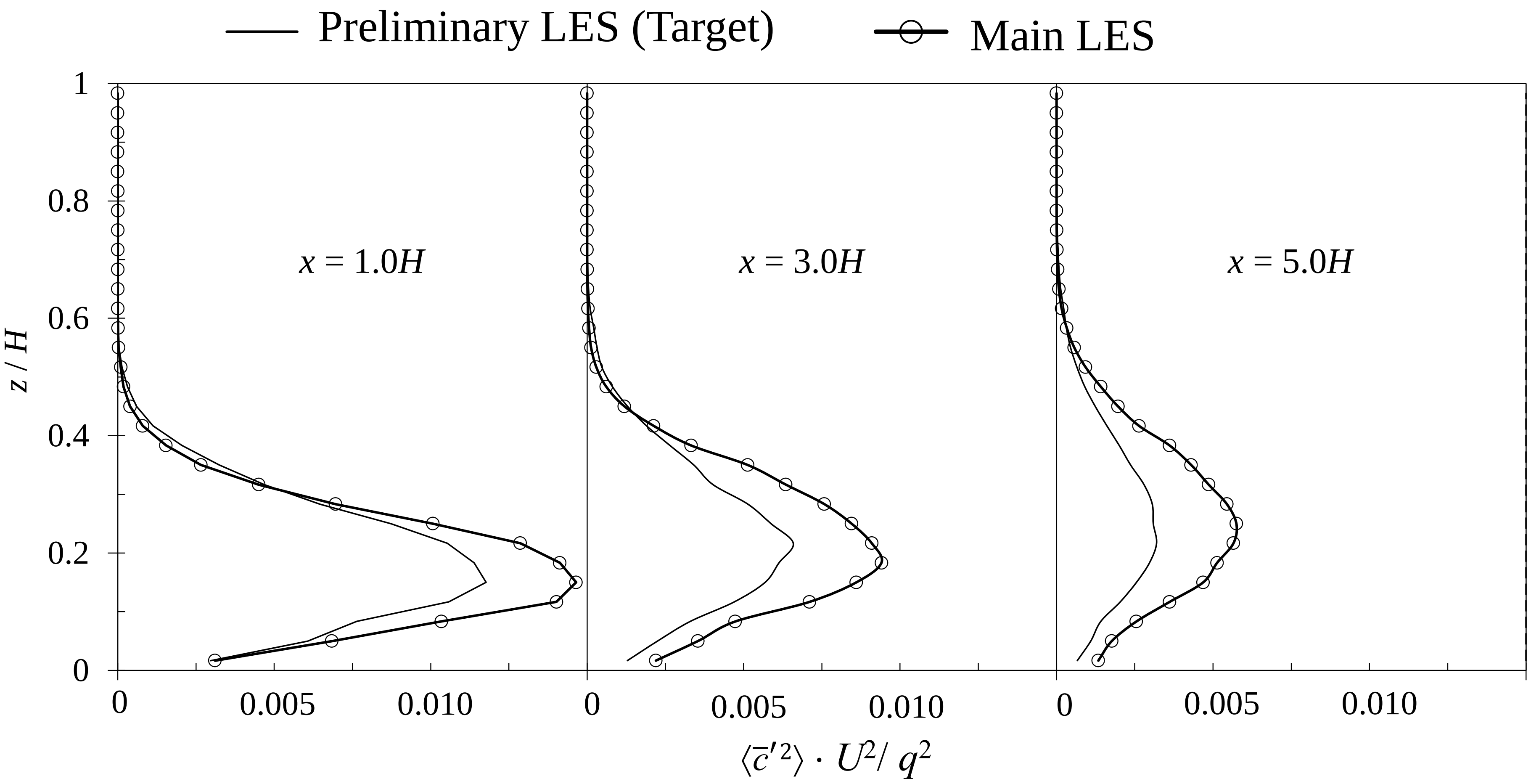
This difference could be attributed to the underestimation of the turbulent diffusion of the passive scalar in the upward direction due to the damping of the turbulent kinetic energy near the inflow boundary.

Streamwise change of $\langle \overline{w}' \overline{c}' \rangle$



The peak values of $\langle \overline{w}' \overline{c}' \rangle$ for both simulations are observed at the same height in each measured line. However, the turbulent flux at x = 1.0H is somewhat smaller than that due to the underestimation of the turbulent kinetic energy.

Streamwise change of $\langle \bar{c}'^2 \rangle$



The result of the main simulation are overestimated for each line although the distribution are similar to the results obtained with the preliminary simulation.

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Conclusions

1. A new method for generating the turbulent fluctuations in wind velocity and scalar such as temperature and contaminants, based on the Cholesky decomposition of the time-averaged turbulent flux tensors of momentum and scalar, was developed.
2. LES computations for a half-channel were carried out to validate the reproducibility of the flow and dispersion fields by applying the artificially generated wind and scalar fluctuations as inflow boundary conditions.

Conclusions

3. By employing a 5×5 non-singular matrix as a turbulent flux tensor matrix, the proposed method can generate simultaneously time series of wind velocity, temperature, and concentration of contaminants and so on.

4. This method can be applied to other artificial generation methods based on the Cholesky decomposition of the Reynolds stress, including the synthetic eddy method (SEM) proposed by Jarrin et al. (2006).

5. Further investigations into the effect of the integral time and length scales of scalar on reproduced dispersion field, as well as the applicability of this method to non-isothermal flow fields should be undertaken. However, this method will couple LES with MMM easily.