A prototype for the evolution of a population of cold pools

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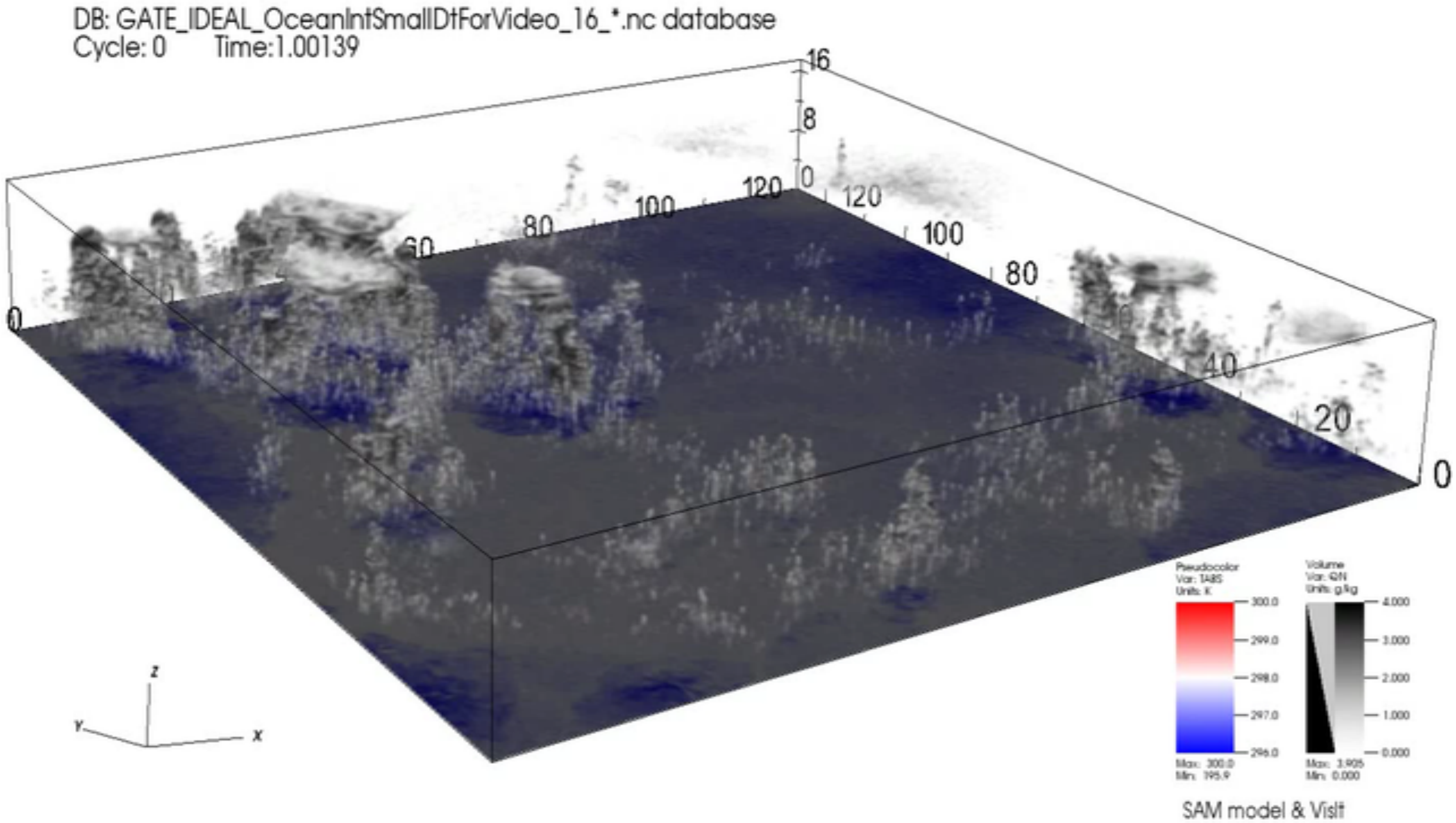
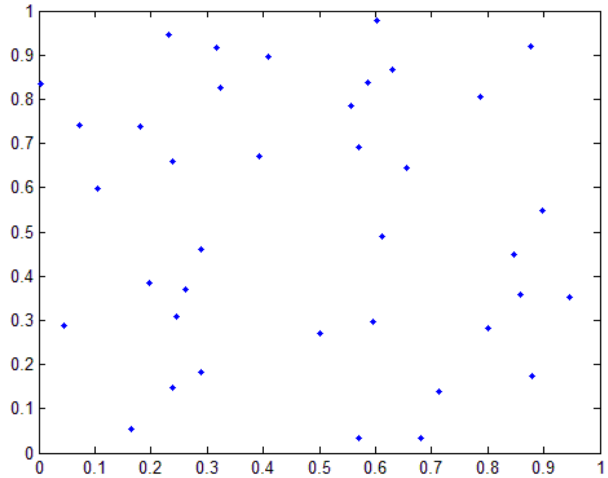


Goal: Give an **evolution equation for the number of CP** present on a domain (or of the **density**), an estimate of the **distribution of their sizes** and an estimate of their **probability of collision**.

H Fundamental hypothesis: the creation of a cold pool is a Poisson point process in 2 dimensions.

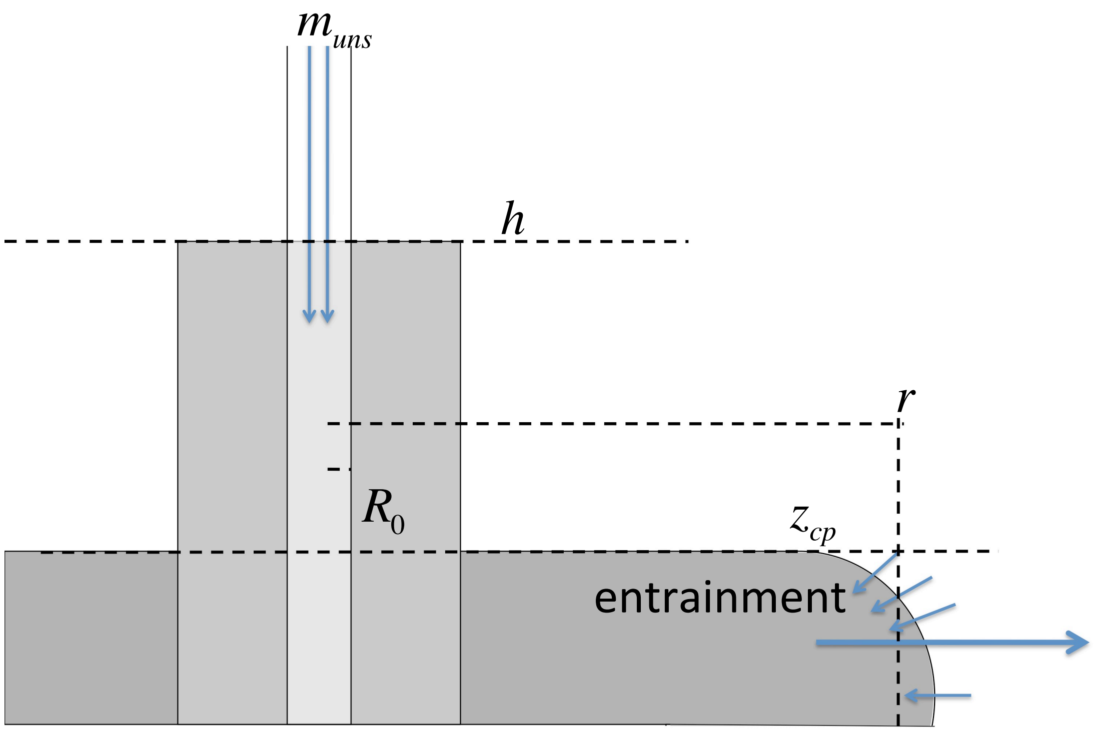


The probability of generating N cold pools within an interval of time *t* is given by the Poisson distribution:

$$P(N,t) = e^{-\Lambda t}\frac{(\Lambda t)^N}{N!}, \ where\ \Lambda = \frac{\lambda \Delta x \Delta y}{L^2}$$

1) Evolution of one cold pool

H Hypothesis : All cold pools are equal.



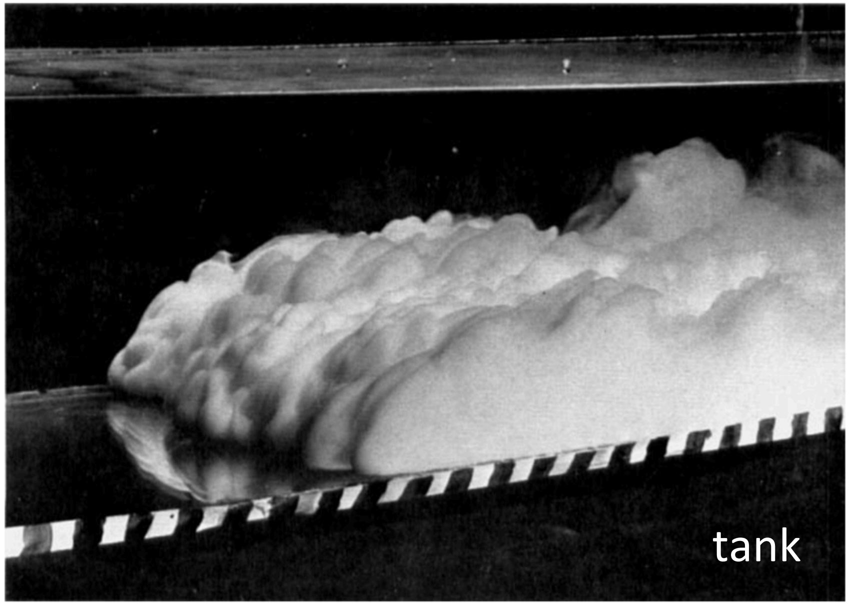
$$c_*(r) = k_* \sqrt{-2\int_0^{z_{cp}} \frac{\theta_{v,\mathrm{cp}} - \theta_{v,\mathrm{env}}}{\theta_{v,\mathrm{env}}} dz}$$

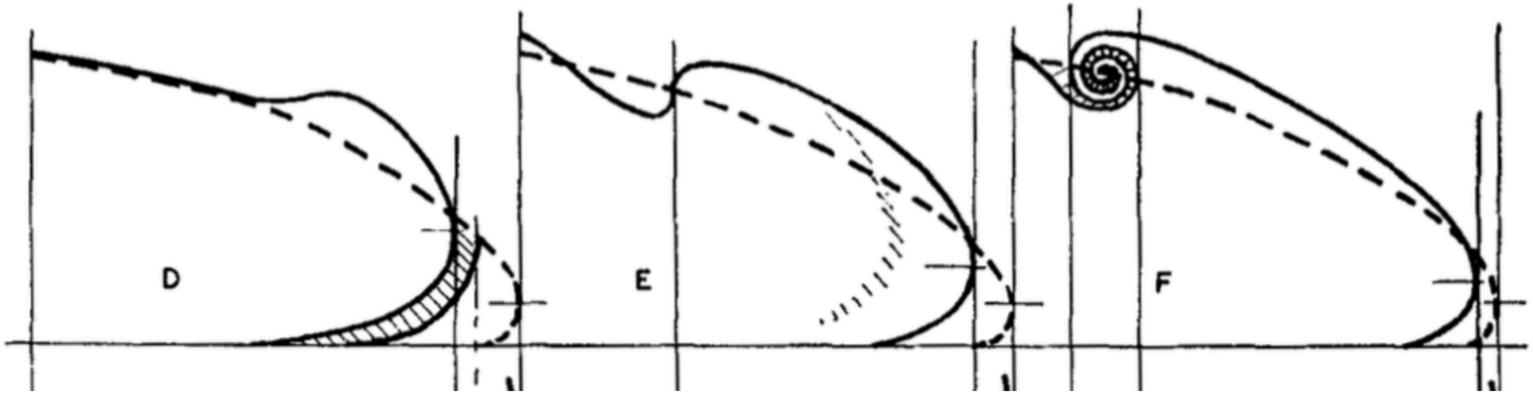
Mass budget:

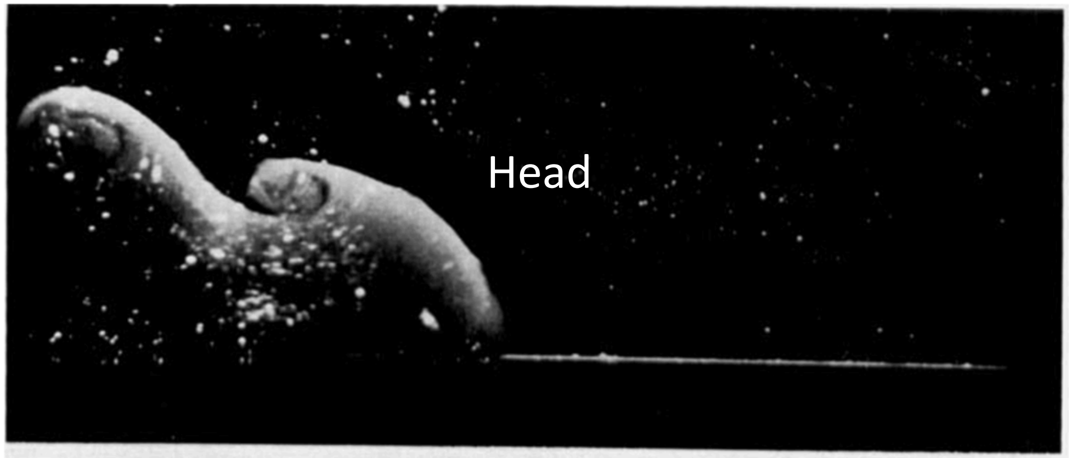
$$\rho \frac{d(\pi r^2 z_{cp})}{dt} = \pi R_0^2 m_{\mathrm{uns}} + \rho \pi^2 r z_{cp} \varepsilon c_*(r) = \frac{M_{\mathrm{uns}}}{\overline{D}} + \rho \pi^2 r z_{cp} \varepsilon c_*(r)$$

Density currents in a tank

Simpson QJRMS 1969

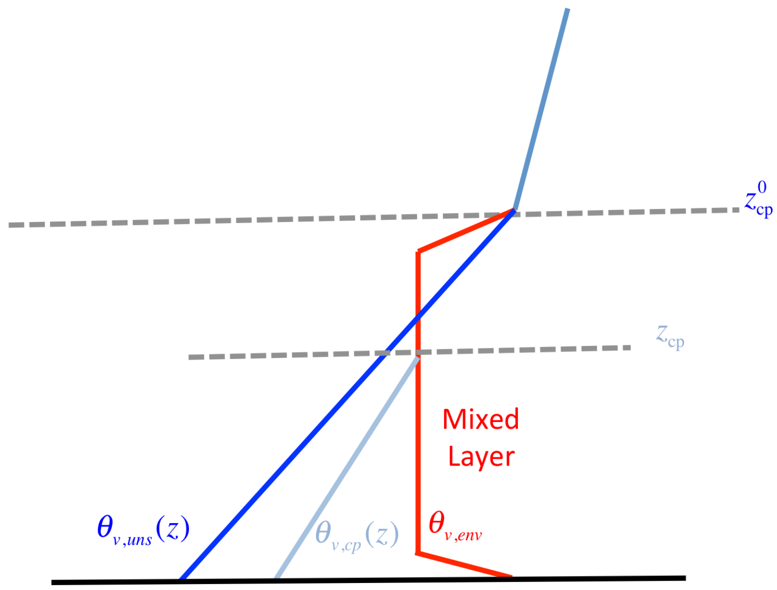






1) Evolution of one cold pool

From the mass balance equation one can work out equations for the CP height and for the temperature



$$\frac{dz_{cp}}{dt} = \left(\frac{R_0}{r}\right)^2 \frac{\dot{m}_{\text{uns}}}{\rho} - (2-\epsilon)\frac{z_{cp}}{r} c_*(r)$$

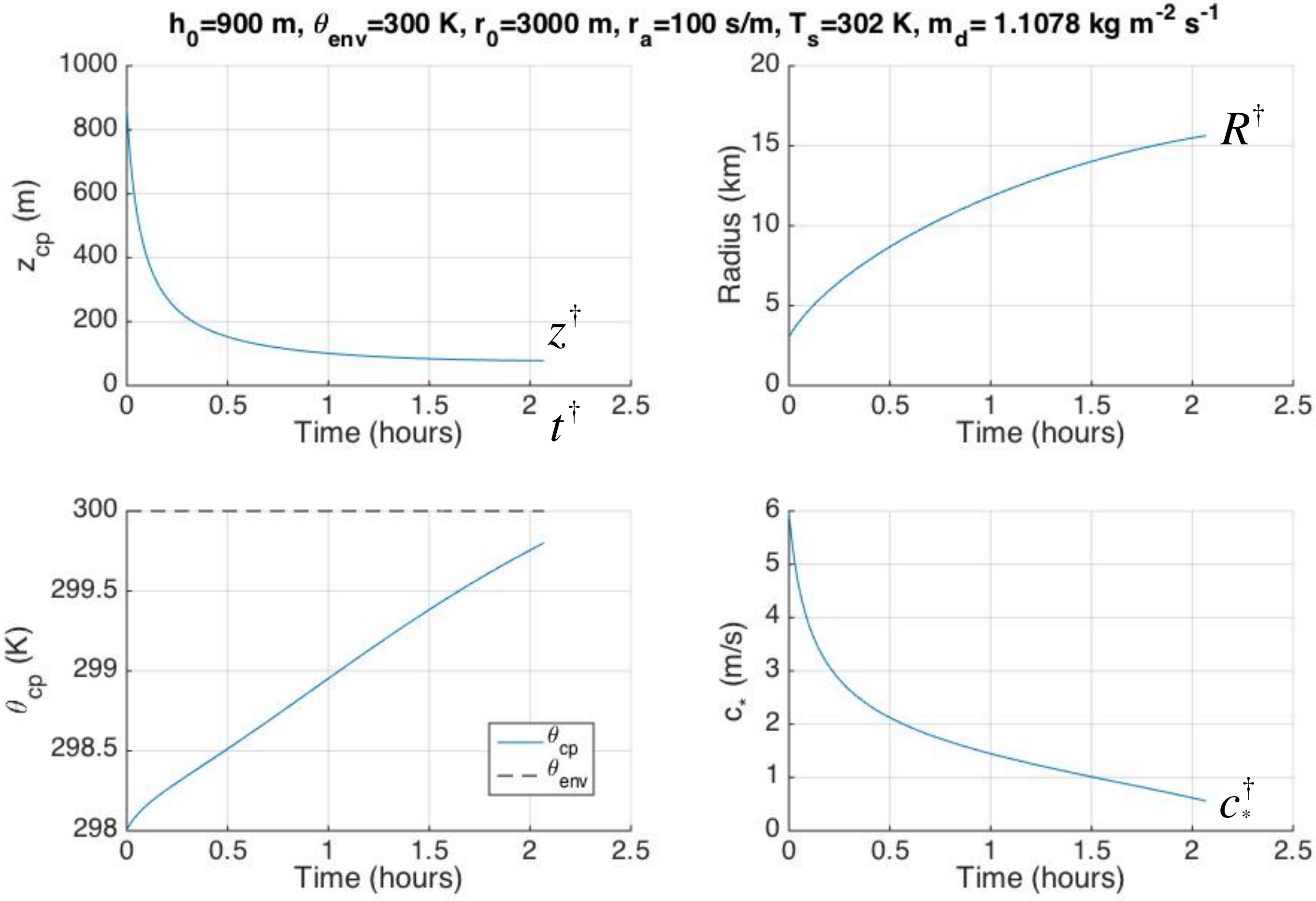
$$\pi r^2 z_{cp} \frac{d\langle\theta_{v,\text{cp}}\rangle}{dt} = \underbrace{\pi R_0^2 \frac{\dot{m}_{\text{uns}}}{\rho}\left(\langle\theta_{v,\text{uns}}\rangle - \langle\theta_{v,\text{cp}}\rangle\right)}_{\text{unsaturated downdraft buoyancy flux}} + \underbrace{\pi r^2 \overline{w'\theta_v'}_{cp}(0)}_{\text{surface heating under cold pool}}$$

$$+ \underbrace{\epsilon \pi r z_{cp} c_*(r)\left(\langle\theta_{v,env}\rangle - \langle\theta_{v,\text{cp}}\rangle\right)}_{\text{entrainment on torus shape}}$$

To these, estimates of the fluxes and of the evolution of the PBL temperature during the lifetime of the CP must be added.

1) Evolution of one cold pool

Example of evolution



2) Cold Pools Density

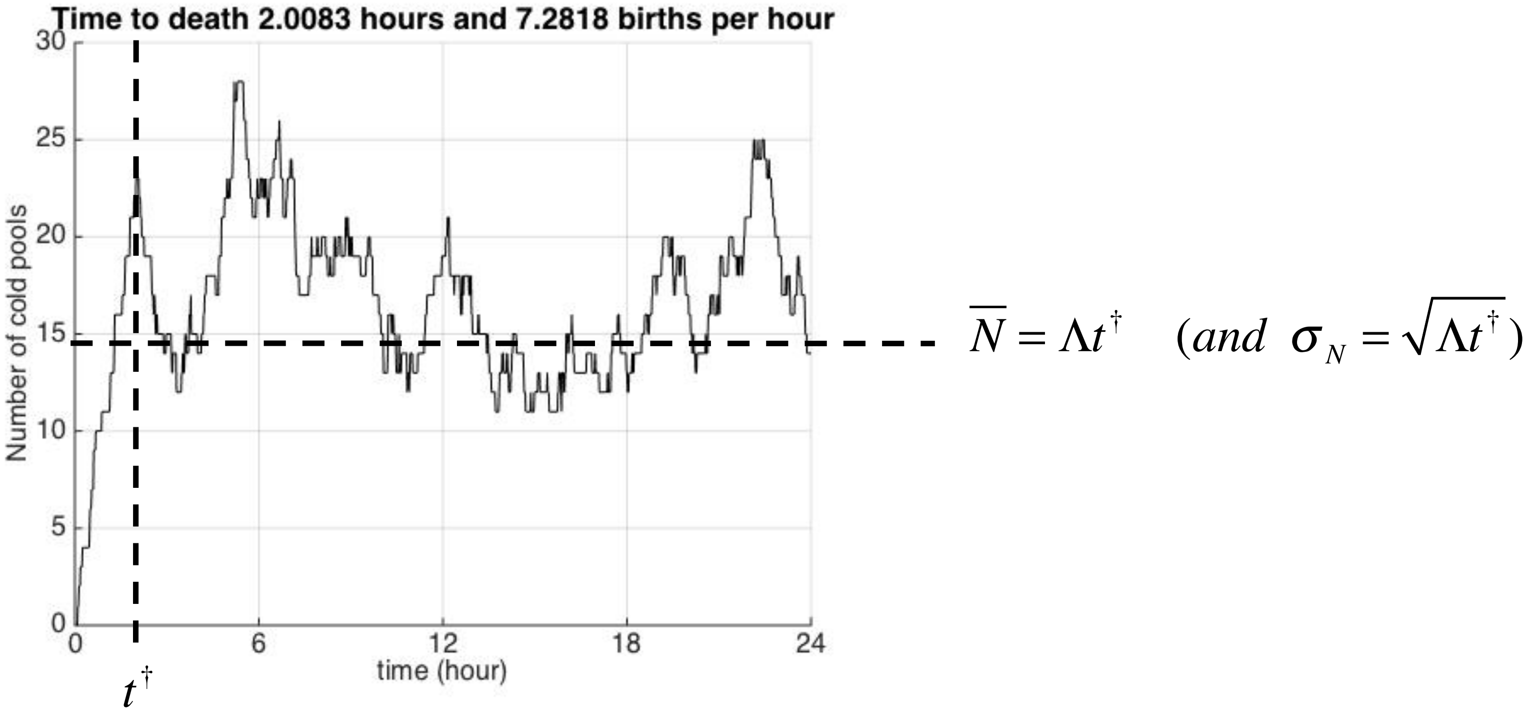
H Hypothesis : the density per radius can be separated in a probability distribution of radiuses and an integrated density:

$$\Delta x \Delta y \; D(r) = \Delta x \Delta y \; \overline{D} \; p(r) = N \; p(r)$$

We hence describe separately the distribution of radius and the evolution of the total number.

2.1) Total Number of Cold Pools

$$P(N,t) = e^{-\Lambda t}\frac{(\Lambda t)^N}{N!},\ where\ \Lambda = \frac{\lambda \Delta x \Delta y}{L^2}$$



The mean number of Cold Pools is given by $\Lambda t^{\dagger}$: this can be used to diagnose Λ from LES.

2.2) Cold Pools Radius distribution

H Hypothesis : cold pools that collide don't mix

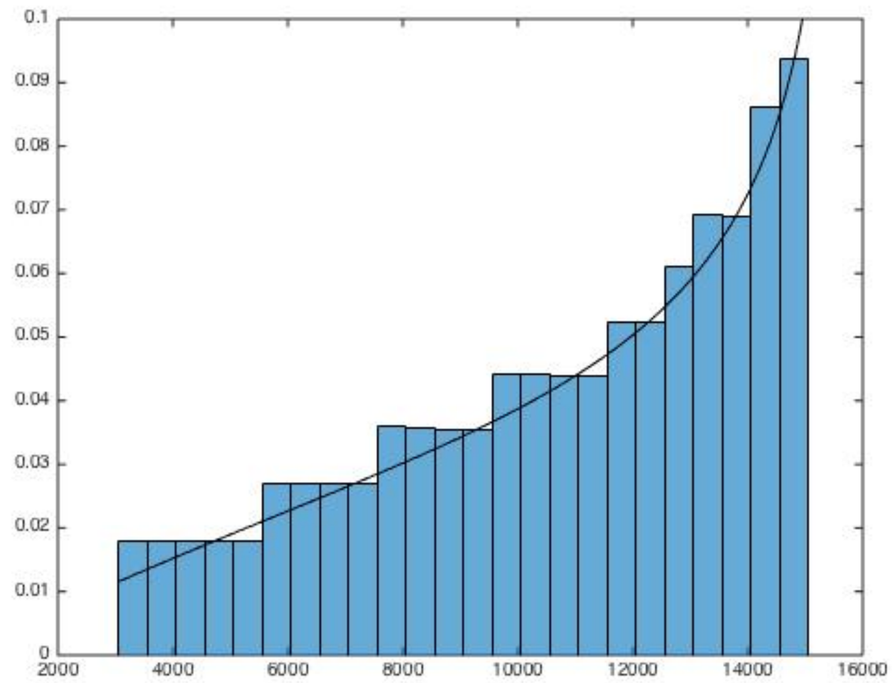
Let's suppose we have 1 generation of CP in a small time period dt: $\frac{dN}{dt} = 1$.
Then:

$$\frac{dN}{dt} = \frac{dN}{dr}\frac{dr}{dt} = \frac{dN}{dr}c^{*}$$, or $$\frac{dN}{dr} = \frac{1}{c^{*}}$$. Hence :

$$\frac{dp(r)}{dr} = \frac{1}{\mathbb{N}}\frac{1}{c^{*}}$$

And the normalization constant is:

$$\mathbb{N} = \int_{R_0}^{R^{\dagger}} \frac{1}{c^{*}} dr = \int_0^{t^{\dagger}} dt = t^{\dagger}$$



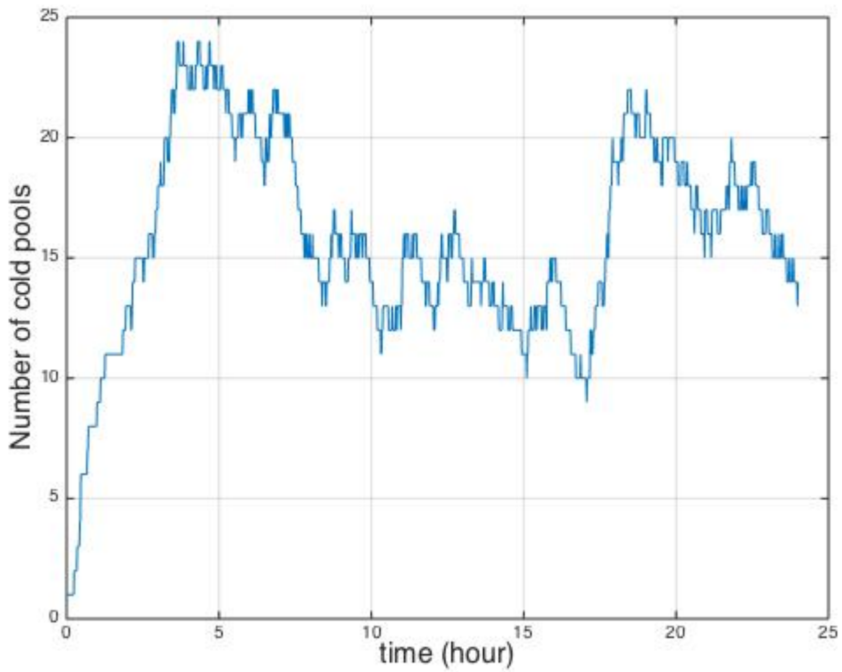
2.1) Total Number of Cold Pools (reprise)

A stochastic equation for the number of cold pools:

$$N(t+dt) = N(t) + N_{in} - N_{out}$$

Where N_{in} is drawn randomly from the Poisson distribution, and

$$N_{out}(t) = \frac{dt}{t^{\dagger}} N + \left(1 - \frac{dt}{t^{\dagger}}\right) N_{out}(t-dt)$$



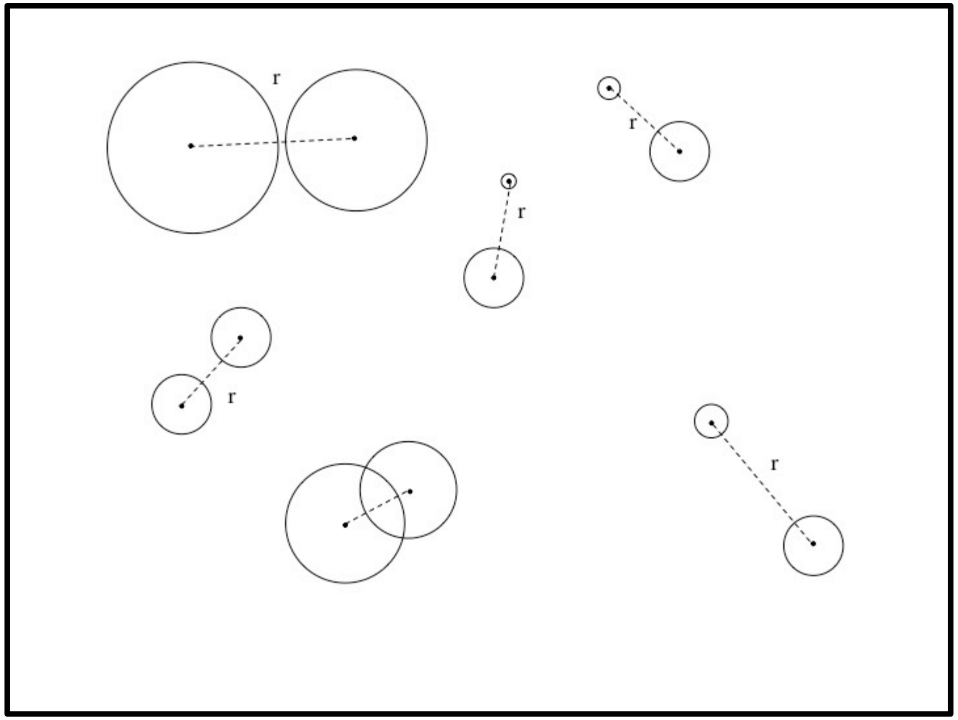
In summary,

at all times, one can compute an estimate of the life expectancy of the cold pools. From that, compute the timestep of the density of cold pools equation.

It would be useful to constrain the Poisson intensity Λ from LES integrations in different configurations of downdraft mass flux and temperature.

At all times, one can also compute the distribution of cold pool size.

3) Directions for a treatment of cold pool collision



In a 2d Poisson point process, the probability distribution of first-neighbour distances is given by:

$$\frac{dD(r)}{dr} = 2\pi\Lambda r\, e^{-\pi\Lambda r^2}.$$

So, the probability that two cold pools of given radius collide is given by

$$p(r_1, r_2) = 2\pi\Lambda \int_0^{r_1+r_2} r\, e^{-\pi\Lambda r^2} dr.$$

Integrating for all cold pool radius, in the measure of p(r), one gets the total probability that any two cold pools collide:

$$\mathrm{P} = 2\pi\Lambda \int_{R_0}^{R^\dagger} \int_{R_0}^{R^\dagger} \int_0^{r_1+r_2} r\, e^{-\pi\Lambda r^2} dr\ p(r_1)\, dr_1\ p(r_2)\, dr_2$$

3) Directions for a treatment of cold pool collision

An approximation to be tested, assuming that all cold pools have the same radius, equal to the mean radius:

$$Number\ of\ collisions = \frac{N}{2}\int_0^{2\bar{r}} 2\pi\Lambda r\, e^{-\pi\Lambda r^2}\, dr,$$

3) Directions for a treatment of cold pool collision

Another important value to be estimated is the length of the collision front. That is where the upward mechanical forcing is maximum and is likely to restart deep convection:

$$\overline{F} = 2\pi\Lambda \int_{R_0}^{r^\dagger} \int_{R_0}^{r^\dagger} \int_0^{r_1+r_2} F(r_1, r_2, r) p(r_1) p(r_2) r e^{-\pi\Lambda r^2} \, dr \, dr_1 \, dr_2$$

With

$$F = \sqrt{4r_1^{\,2} - \frac{2}{r}\left(r_1^{\,2} - r_2^{\,2} + r^2\right)},$$

Or, assuming again that all CPs have the radius equal to the mean radius:

$$F = \sqrt{4\overline{r}^{\,2} - r^2}$$