Improving coupled model solution mathematical consistency through data assimilation.

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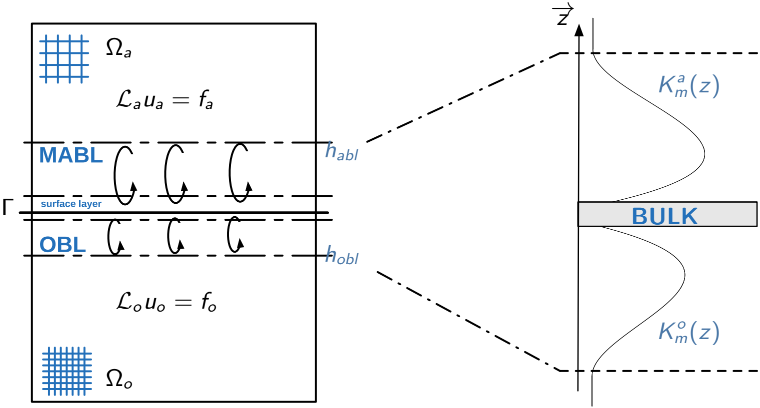
Coupled Data Assimilation Workshop

Coupled Data Asimilation: an opportunity

OA coupling is a complex matter with many sources of uncertainties

- time/space non-confomity
- interfaces may actually not be represented by any component
- multi physics with different characteristics.
- highly parameterised interface (Bulk formulae)
- coupling methods
- ...

Some of theses uncertainties are unavoidable, some others are linked to the way we implement things.
Coupled DA is an opportunity to account for or reduce them

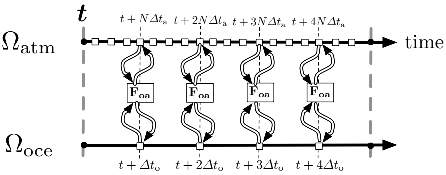


- K_m parameterization in the boundary layers
- $\boldsymbol{F}_{oa} = (\tau, Q_{net})$ is the interface flux

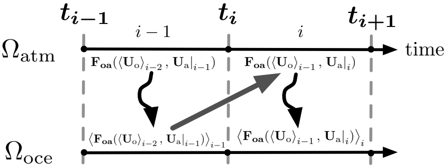
At the Interface Γ :

$$\mathcal{G}_o u_o = \mathcal{G}_a u_a$$

$$\mathcal{F}_a u_a = \mathcal{F}_o u_o$$



SYNCHRONOUS



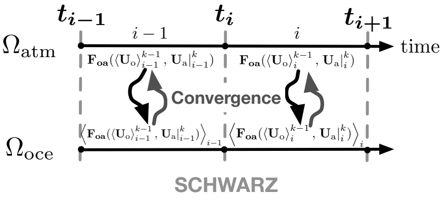
ASYNCHRONOUS

- Synchronous method.
  - Aliasing errors
  - Synchronicity issues
  - Physics-dynamics inconsistency error

- Asynchronous method:
  - Does not solve the original problem

Possible solutions

- Monolithic approach (Type 1 coupling in P.Laloyaux's nomenclature)
- Iterative method to solve the coupling problem



Considering :

- $u_0 \in H^1(\Omega_a \cup \Omega_o)$ the initial condition
- k the iteration number
- $u_a^0(0,t)$ the *first-guess*

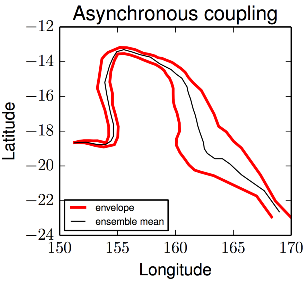
The SWR algorithm reads :

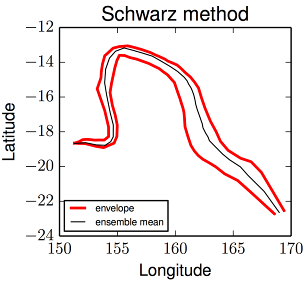
$$\begin{cases} \mathcal{L}_o u_o^k = f_o & \text{on } \Omega_o \times T_i \\ u_o^k(z,0) = u_0(z) & z \in \Omega_o \\ \mathcal{G}_o u_o^k = \mathcal{G}_a u_a^k & \text{on } \Gamma \times T_i \end{cases} \qquad \begin{cases} \mathcal{L}_a u_a^k = f_a & \text{on } \Omega_a \times T_i \\ u_a^k(z,0) = u_0(z) & z \in \Omega_a \\ \mathcal{F}_a u_a^k = \mathcal{F}_o u_o^{k-1} & \text{on } \Gamma \times T_i \end{cases}$$

where $T_i = [t_i; t_{i+1}]$

- At convergence, it provides a flux consistent solution : $\mathcal{F}_a u_a = \mathcal{F}_o u_o$ and $\mathcal{G}_o u_o = \mathcal{G}_a u_a$ on $\Gamma \times T_i$

Coupling methods
Why does it matter





Hurricane Erica's trajectory and ensemble spread
18 members of WRF/ROMS, generated through perturbations of initial conditions and coupling frequency (lemarié et al. 2014))

Main drawbacks :

- This is an iterative method
- Convergence speed greatly depends on $\mathcal{F}_d$, $\mathcal{G}_d$ and $u_a^0(0,t)$ $(d = a, o)$

Advantages :

- This is a non-intrusive coupling method
- At convergence, it provides a strongly coupled solution

Starting point of Rémi's PhD, in the framework of a variational system

- Can we improve the boundary conditions to accelerate the SWR convergence?
- Take benefit of the minimisation iterations for the SWR ones

Fully Iterative Method (FIM)

- $\mathbf{x}_0 = u_0(z),\ z \in \Omega = \Omega_a \cup \Omega_o$ is the controlled state vector
- $\mathbf{x}^{\mathrm{cvg}} = (u_a^{k_{\mathrm{cvg}}}, u_o^{k_{\mathrm{cvg}}})^T$ is the converge solution of the SWR algorithm : k_{cvg} iterations
- The first-guess u_a^0 in the SWR algorithm is updated after each minimisation iteration

$$J_{FIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_{t_i}^{t_{i+1}} \left\langle \mathbf{y} - H\left(\mathbf{x}^{\mathrm{cvg}}\right), \mathbf{R}^{-1}(\mathbf{y} - H\left(\mathbf{x}^{\mathrm{cvg}}\right)) \right\rangle_{\Omega} \mathrm{dt}$$

- The solution provided is ~~strongly~~ ~~fully~~ insanely coupled
- It requires the adjoint of the coupling
- It possibly requires a large number of Schwarz iterations

Truncated iterative method (TIM)

- $\mathbf{x}_0 = (u_0(z), \underline{u_o^0(0,t)})^T, \ z \in \Omega \setminus \Gamma$
- The Schwarz iterations are truncated at $k_{\max} < k_{\mathrm{cvg}}$ iterations
- $\mathbf{x}^{\mathrm{max}} = (u_a^{k_{\max}}, u_o^{k_{\max}})^T$
- Extended cost function :

$$J^s = \alpha_{\mathcal{F}} \|\mathcal{F}_a u_a^{k_{\max}}(0,t) - \mathcal{F}_o u_o^{k_{\max}}(0,t)\|^2_{T_i} + \alpha_{\mathcal{G}} \|\mathcal{G}_a u_a^{k_{\max}}(0,t) - \mathcal{G}_o u_o^{k_{\max}}(0,t)\|^2_{T_i}$$

with $\|a\|^2_\Sigma = \langle a, a \rangle_\Sigma$

$$J_{TIM}(\mathbf{x}_0) = J^b(\mathbf{x}_0) + \int_{t_i}^{t_{i+1}} \left\langle \mathbf{y} - H(\mathbf{x}^{\mathrm{max}}), \mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x}^{\mathrm{max}})) \right\rangle_\Omega \mathrm{dt} + \boxed{J^s}$$

- The solution provided is quasi-strongly coupled
- It requires the adjoint of the coupling
- It requires fewer number of Schwarz iterations than the FIM

Weakly Interfaced Models (WIM)

- $\mathbf{x}_0 = (\mathbf{x}_{0,a}, \mathbf{x}_{0,o})^T$ with $\mathbf{x}_{0,d} = \left(u_0|_{z\in\Omega_d}, u_d^0(0,t) \right)$
- The direct coupling between both models is suppressed
- Models are coupled during the assimilation process

$$J_{WIM}(\mathbf{x}_0) = \left\{ \sum_{d=a,o} \left(J^b(\mathbf{x}_{0,d}) + J^o(\mathbf{x}_{0,d}) \right) \right\} + J^s$$

- The solution provided is weakly coupled (as coupling is a weak constraint)
- It requires only the adjoints of the uncoupled models
- There is no coupling iterations

Considered schemes - Summary

Algo	Control vector	# of coupling iterations	extended cost function	Adjoint of the coupling	Coupling
FIM	$(u_0(z))$	k_{cvg}	no	yes	strong
TIM	$(u_0(z), u_o^0)^T$	$k_{\max}$	possibly	yes	$\sim$strong
WIM	$(u_0(z), u_a^0, u_o^0)^T$	1	yes	no	weak

Table: Overview of the properties of the coupled variational DA methods described

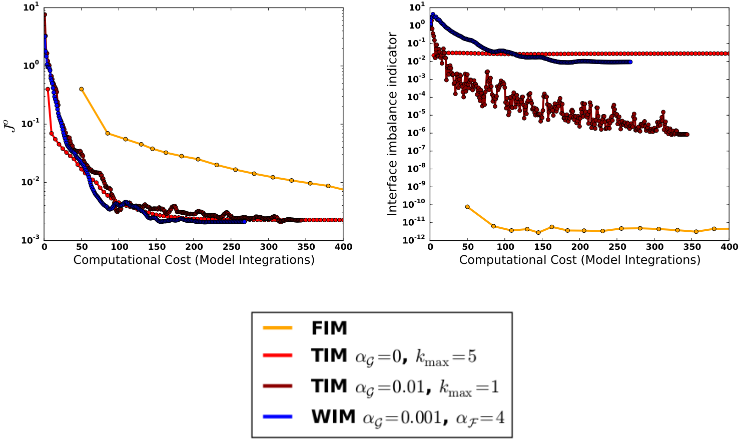
Previous algorithms are applied on a **1D linear diffusion problem**. Let us consider $(d = a, o)$:

- $\mathcal{L}_d = \partial_t + \nu_d \partial_z^2$
- $\nu_a \neq \nu_o$ the diffusion coefficients
- $\mathcal{G}_d = \nu_d \partial_z$ and $\mathcal{F}_d = \mathrm{Id}$ the interface operators on Γ
- f_d the second member such that the analytical solution is $u_d^\star(z,t) = \frac{U_0}{4} e^{-\frac{|z|}{\alpha_d}} \left\{3 + \cos^2\left(\frac{3\pi t}{\tau}\right)\right\}$ on $\Omega_d \times T_i$

Application to a 1D diffusion problem

Our simple coupled system : two coupled 1D diffusion equations - results

Algo	$\alpha_{\mathcal{F}}$	$\alpha_{\mathcal{G}}$	$k_{\max}$	# of minimisation iterations	# of models runs	Interface imbalance indicator	RMSE in °C
FIM	-	-	k_{cvg}	58	1169	$3.69\ 10^{-12}$	0.220
TIM	0	-	k_{cvg}	48	2016	$5.63\ 10^{-12}$	0.220
TIM	0	-	5	245	1225	$2.91\ 10^{-2}$	0.216
TIM	0	-	2	1518	3036	3.77	0.272
TIM	0.01	-	2	425	850	$9.89\ 10^{-7}$	0.217
TIM	**0.01**	-	**1**	**344**	**344**	$\mathbf{8.38\ 10^{-7}}$	**0.215**
WIM	0.01	40	1	2957	2957	$1.40\ 10^{-4}$	0.231
WIM	**0.001**	**4**	**1**	**268**	**268**	$\mathbf{9.38\ 10^{-3}}$	**0.240**
WIM	0.0001	0.4	1	742	742	$3.29\ 10^{-1}$	0.327
Uncoupled	0	0	1	101	101	29.0	1.717



$\mathcal{J}^o$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0 50 100 150 200 250 300 350 400
Computational Cost (Model Integrations)
Interface imbalance indicator
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
0 50 100 150 200 250 300 350 400
Computational Cost (Model Integrations)
FIM
TIM $\alpha_{\mathcal{G}}=0$, $k_{\max}=5$
TIM $\alpha_{\mathcal{G}}=0.01$, $k_{\max}=1$
WIM $\alpha_{\mathcal{G}}=0.001$, $\alpha_{\mathcal{F}}=4$

Final comments

It is difficult to draw a clear conclusion from such a simplistic testcase but

- The way models are coupled should not be overlooked
- FIM and even TIM are probably too extreme
- but controlling (as we saw this morning) and/or penalising the interface mismatch could be a step toward stronger coupling

More work for Rémi:

- more in depth theoretical study on convergence
- Apply these algorithms to a more realistic coupled SCM (Ocean/ABL, currently being implemented within OOPS)
- look into optimized interface conditions for SWR

In parallel:

- extend this work to ensemble smoother