# OPTIMIZATION AND BAYESIAN APPROACHES FOR MODEL CALIBRATION

# APPLICATION TO OIL AND GAS FIELD MANAGEMENT

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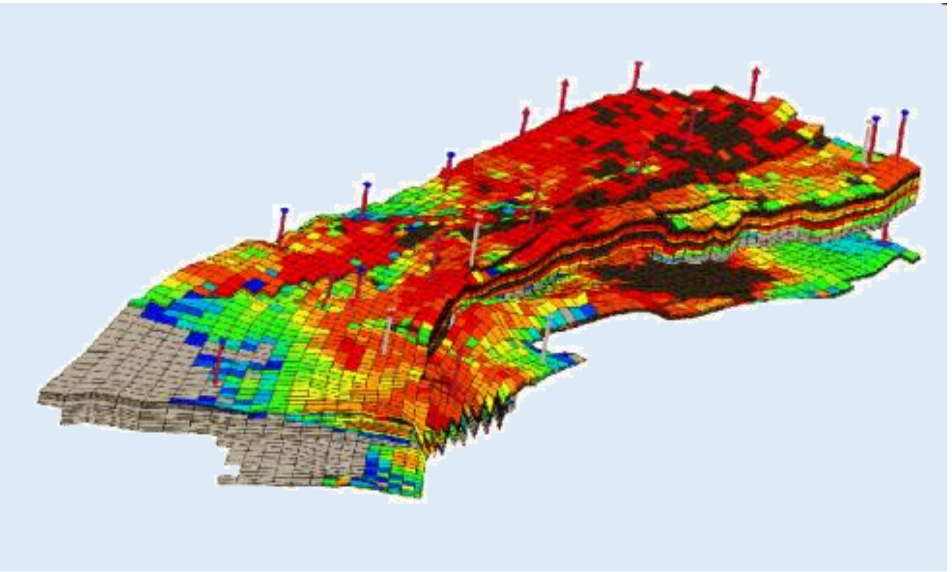
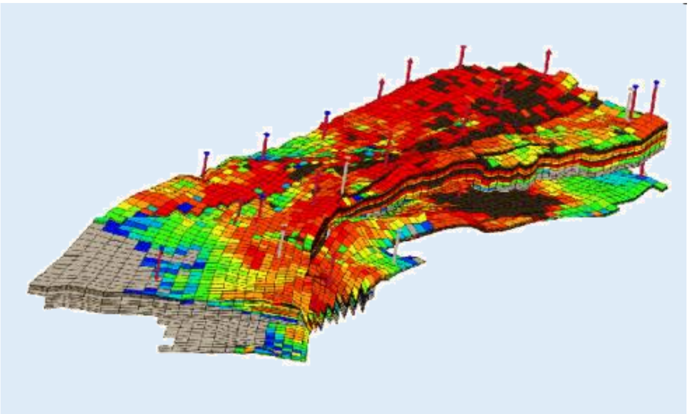
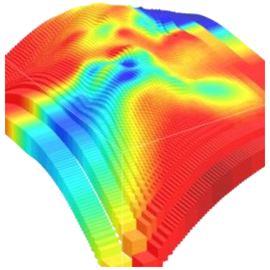


# OIL AND GAS FIELD MANAGEMENT

- We are interested in the **forecast** of oil/gas production and Bottom Hole Pressure (BHP) of a petroleum reservoir





- We have at our disposal
  - **Measured Data** on the field (i.e. production until a given time)
  - A **parametrized simulator** of the petroleum reservoir

# OPTIMIZATION AND BAYESIAN APPROACHES FOR MODEL CALIBRATION AND PREDICTION

- **Bayesian calibration + prediction via propagation**

  *Quantify what information my uncertain observations give me on my calibration parameters*

  - Estimate **posterior distribution** of a reservoir model parameters from history data

  *Distribution/Sampling of* $New\ observation\ knowing\ old\ observations$

  - **Propagate** the model **parameters uncertainties on predictions**

- **Extreme scenario prediction**
  - Minimize/Maximize (forecast production) subject to History Matching constraint

# DATA AND ERRORS

- My $n$ **real physical responses** of the experiment under given operational conditions $x_{op}^{i}$

$$d_i^{real} \quad (i = 1, \dots, n)$$ → Unseen

- My $n$ **measured observations** under operational conditions $x_{op}^{i}$ :

$$d_i^{obs} \quad (i = 1, \dots, n)$$ → Example: oil production history measured on the field

- The **measurement errors** on my observations:

$$\epsilon_i^{m}$$ → unknown

**My Data** : $x_{op}^{i}, d_i^{obs} \quad (i = 1, \dots, n)$

# MODELLING DATA

- **« Observational » model**:

$$d_i^{obs} = d_i^{real} + \epsilon_i^m$$

- My simulator responses with **unknown optimal calibration** parameters $x^*$ evaluated at operationnal conditions $x_{op}^i$:

$$G\left(x_{op}^i, x^*\right)$$

- My **simulator mimic** exactly, or at best, **the real physical phenomenon**:

$$d_i^{real} = G\left(x_{op}^i, x^*\right) + \epsilon_i^{sim}$$

- **« Observational-simulated » model**:

$$d_i^{obs} = G\left(x_{op}^i, x^*\right) + \epsilon_i$$

with $\epsilon_i = \epsilon_i^m + \epsilon_i^{sim}$

# STATISTICAL MODELLING

- « Observational-simulated » model

$$d_i^{obs} = G(x_{op}^i, x^*) + \epsilon_i$$

- **Statistical modelling:** $D^{obs} = G(x_{op}, X) + \epsilon$ → Random variables

- What we do not know and that we are going to model are : $\epsilon$ and $X$
  Choice of
  - **distribution** of the **error** $\epsilon$
  - A **prior distribution** on the calibration **parameters** $X$

- **Knowing my observations what is the distribution of my calibration parameters**

# BAYESIAN CALIBRATION AND PREDICTION

- Bayesian **Calibration**

$$p\left(X = x \middle| D_i^{obs} = d_i^{obs}\right) \alpha \; p\left(D_i^{obs} = d_i^{obs} \middle| X = x\right) p(X = x)$$

Likelihood

prior

posterior

- Bayesian **Prediction model**

$$D_{new} = G(x_{new}, X) + \epsilon$$

- **Theoretical prediction distribution**

$$p\left(D_{new} \middle| D_i^{obs} = d_i^{obs}\right) = \int p(D_{new} | X = x) p\left(X = x \middle| D_i^{obs} = d_i^{obs}\right) dx$$

- **Propagation**

MCMC* Sample of $X_i$ from $p\left(x \middle| D_i^{obs} = d_i^{obs}\right)$ → Propagate $D_{new}^i = G(x_{new}, X_i) + \epsilon_i$ → Calculate statistics from $D_{new}^i$

* : Monte Carlo Markov Chain

# BAYESIAN CALIBRATION AND PREDICTION

- Problem :
  - MCMC procedure requires a **large number of evaluation of the simulator** for different values of the parameters
  - **Simulator** is **expensive** to evaluate

- Solution : use a **surrogate** model but of what ?

  - In MCMC we need to **evaluate the posterior** distribution :

$$p\left(X|D_i^{obs} = d_i^{obs}\right) \alpha \; p\left(D_i^{obs} = d_i^{obs}|X\right)p(X)$$

  - If **Gaussian** hypothesis (i.i.d) on the **error** $\epsilon$ then

$$p\left(D_i^{obs} = d_i^{obs}|X = x\right) = Cexp\left(-\sum_i \frac{\left(G\left(x_{opt}^i, x\right) - d_i^{obs}\right)^2}{\sigma_i}\right)$$

$$= Cexp(-F(x))$$

Option 1
Build surrogate

Option 2
Build surrogate

# SURROGATE

- Sample the posterior distribution via Markov Chain Monte Carlo coupled with an **adaptive <u>Kriging</u>** (Gaussian process regression) surrogate:

  - **Standard approach**: surrogate of $F(x)$ from a set of points $(x_i, F(x_i))$ build a predictive surrogate $\hat{F}$

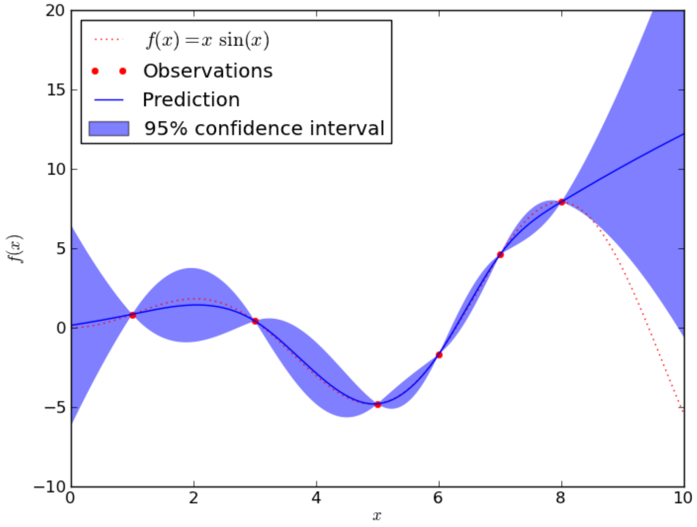
  - **Adaptive approach**: surrogate adaptively improved with iterative choice of design points $x_i$ where
    - the **variance** of the surrogate **prediction** is **maximum**
    - the **Expected Improvement** criterion is **maximum** to emphasis good surrogate prediction where $F(x)$ is small: avoid negative approximation values of a positive function

$$\boldsymbol{Gaussian\ Process:}\ \hat{F} \sim \boldsymbol{N(m, \Sigma)}$$
$$\boldsymbol{Gaussian\ Random\ Variable\ (RV):}\ \hat{F}(\boldsymbol{x}) \sim \boldsymbol{N(m(x), \sigma(x))}$$

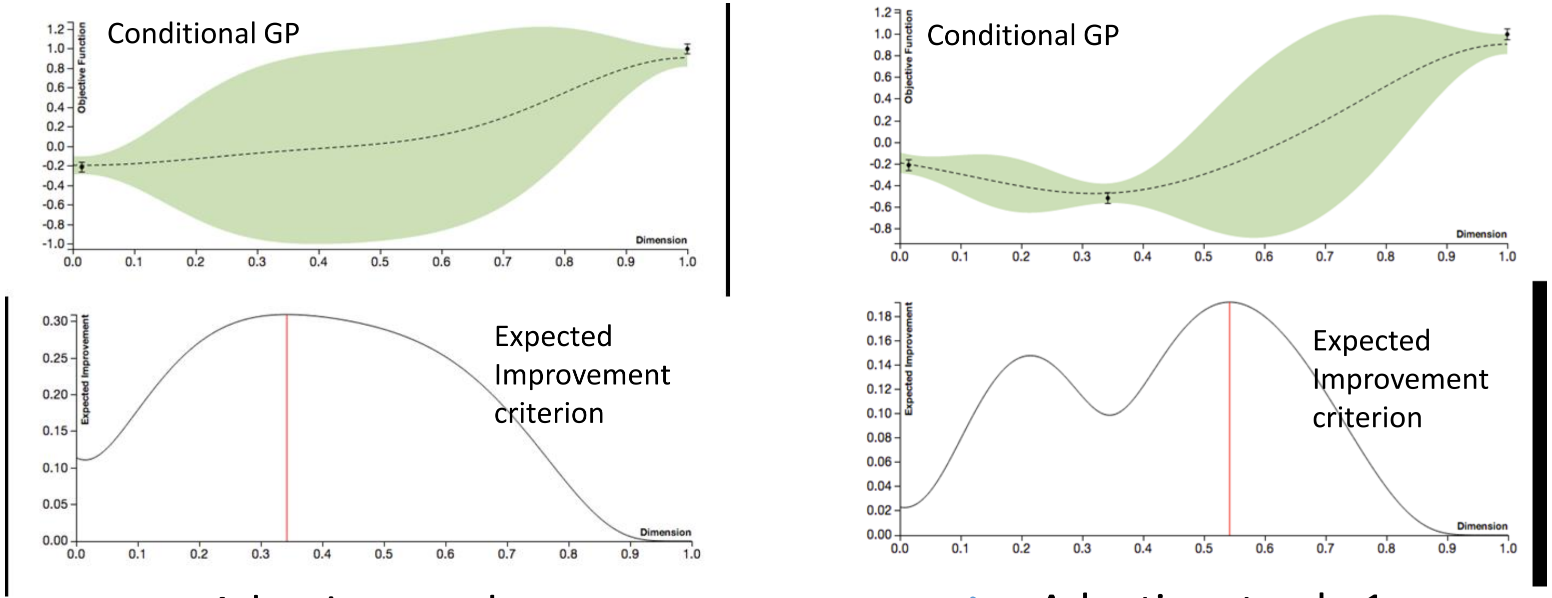
# GAUSSIAN PROCESS REGRESSION A.K.A. KRIGING

$$\textit{Gaussian Process}: \ \hat{F} \sim N(m, \Sigma)$$
$$\textit{Gaussian RV}: \ \hat{F}(x) \sim N(m(x), \sigma(x))$$

- The model is assumed to be a **realization of a Gaussian Process** (GP) with parametric prior mean function and a given covariance function.
- The surrogate is given by the **mean of the GP conditionally to the observations** $\hat{F}$




Conditional GP




Adaptive step k → Adaptive step k+1

# BAYESIAN APPROACH

$$\boldsymbol{F(x) \approx \hat{F}(x) \sim N\big(m(x), \sigma(x)\big)}$$

MCMC Sample of $X_i$ from $p\left(x|D_i^{obs} = d_i^{obs}\right)$

→ Propagate $D_{new}^i = G(x_{new}, X_i) + \epsilon_i$

→ Calculate statistics from $D_{new}^i$

« Plugged-in » posterior distribution

$Cexp(-m(x)\ )$ x $p(x)$

- Some remarks
  - Need sufficiently accurate surrogate $m$ on all the parameters space or take into account the surrogate model error in the calibration
  - Negative values of $m$ involves bad behavior of MCMC sampling
  - Propagation step requires running the simulator a non-negligible number of times

    (For this application: at least 500 simulations needed)
- Some proposals:
  - Apply MCMC with **surrogates of residuals**
  - **Constraint surrogate** to be positive : constraint Gaussian Process
  - or **build surrogate $\tilde{F}$ of $\sqrt{F}$** then square and input in the likelihood
  - **Include Kriging error** in the procedure

$$F(x) \approx \widehat{F}(x) \sim N\big(m(x), \sigma(x)\big)$$
$$\sqrt{F(x)} \approx \widetilde{F}(x) \sim N(\widetilde{m}(x), \widetilde{\sigma}(x))$$

- Accounting for the Kriging error in the posterior distribution

| Without Kriging | | With Kriging (Random) | | With plugged-in Kriging predictor (mean) |
|---|---|---|---|---|
| $e^{-F(x)}p(x)$ | → | $U(x) := e^{-\hat{F}(x)}p(x)$ | → | $e^{-m(x)}p(x)$ |
| $e^{-\sqrt{F(x)}^2}p(x)$ | → | $V(x) := e^{-\tilde{F}^2(x)}p(x)$ | → | $e^{-\tilde{m}^2(x)}p(x)$ |

↓

What are the distributions of U(x) and V(x) ?

# BAYESIAN APPROACH

$$\widehat{\boldsymbol{F}}(\boldsymbol{x}) \sim \boldsymbol{N}\big(\boldsymbol{m}(\boldsymbol{x}), \boldsymbol{\sigma}(\boldsymbol{x})\big)$$
$$\widetilde{\boldsymbol{F}}(\boldsymbol{x}) \sim \boldsymbol{N}(\widetilde{\boldsymbol{m}}(\boldsymbol{x}), \widetilde{\boldsymbol{\sigma}}(\boldsymbol{x}))$$

- Accounting for the Kriging error of $F(x)$ in the posterior distribution (By using log-normal distribution)

$$Expectation\big(U(x)\big) = e^{-(m(x)-\sigma^2(x)/2)}p(x)$$

$$Mediane\big(U(x)\big) = e^{-m(x)}p(x)$$

$$Mode\big(U(x)\big) = e^{-(m(x)+\sigma^2(x))}p(x)$$

- Accounting for the Kriging error of $\sqrt{F}$ in the posterior distribution (By using Chi-square distribution)

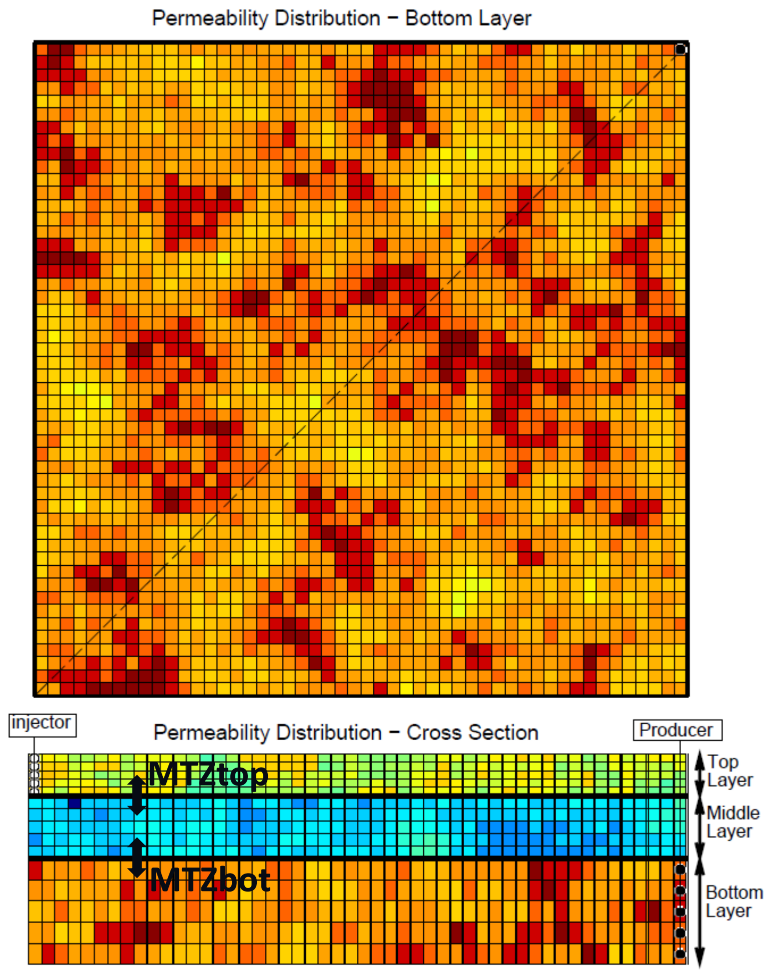
$$e^{-(\widetilde{m}^2(x)+\widetilde{\sigma}^2(x))}p(x)$$

# TEST CASE : SPE1

- 1 gas injector well
- 1 producer well
- Grid : 50x50x15  3 layers
- Permeabilities modeled by spherical variograms for each layer
  1 parameter per layer
  *Kbot_mean, Kmid_mean, Ktop_mean*
- 2 Factors for vertical transmissivities
  for inter-layer permeability barriers
  *MTZbot, MTZtop*
- 2 Well productivity indexes
  *MPI_inj, MPI_prod*

➢ **7 parameters to calibrated**

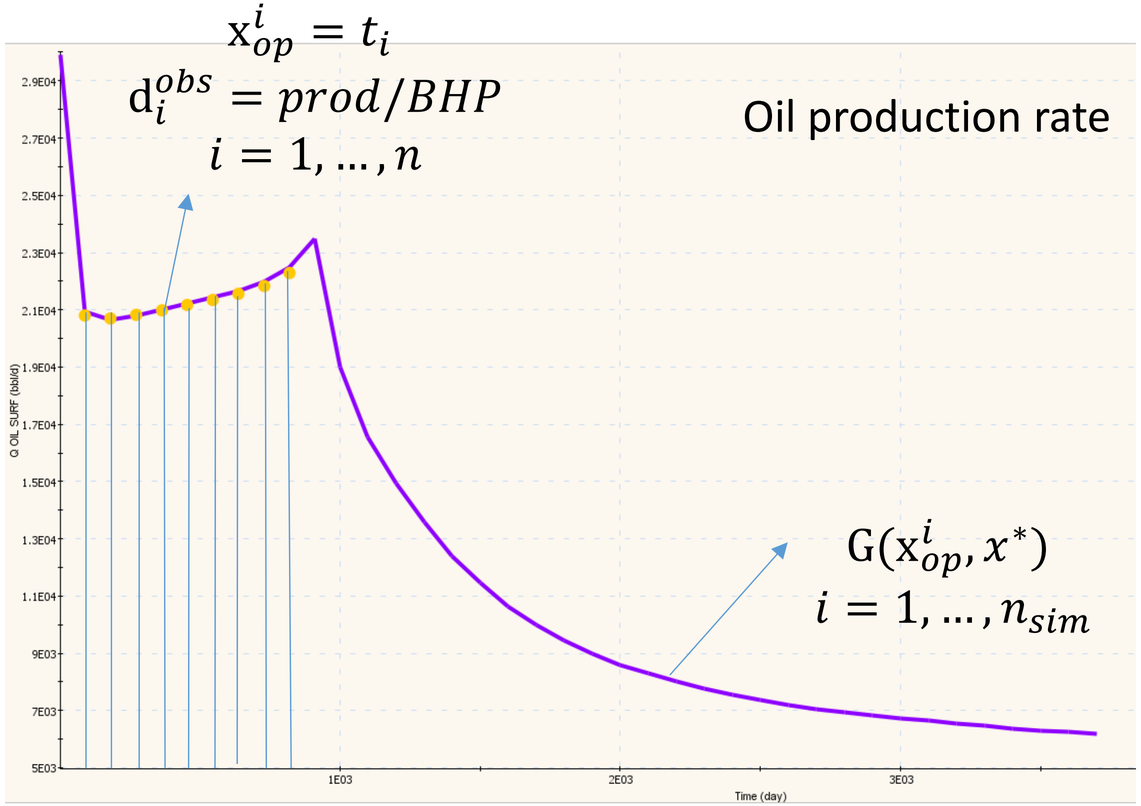


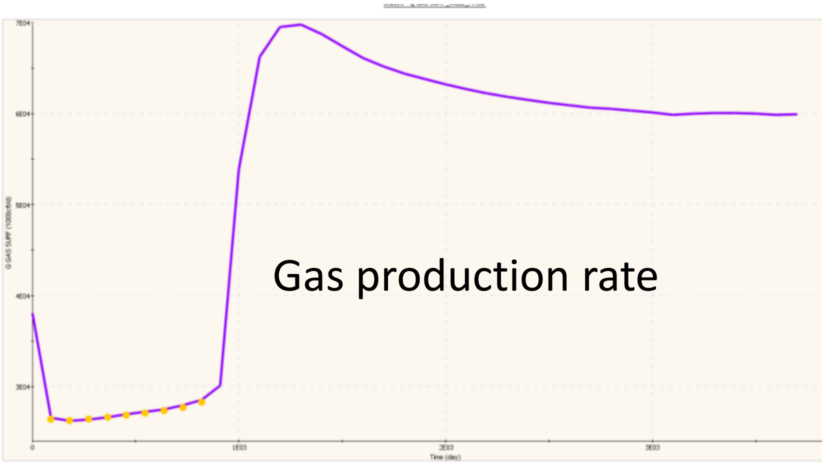

*Roggero and Guerillot, 1996*

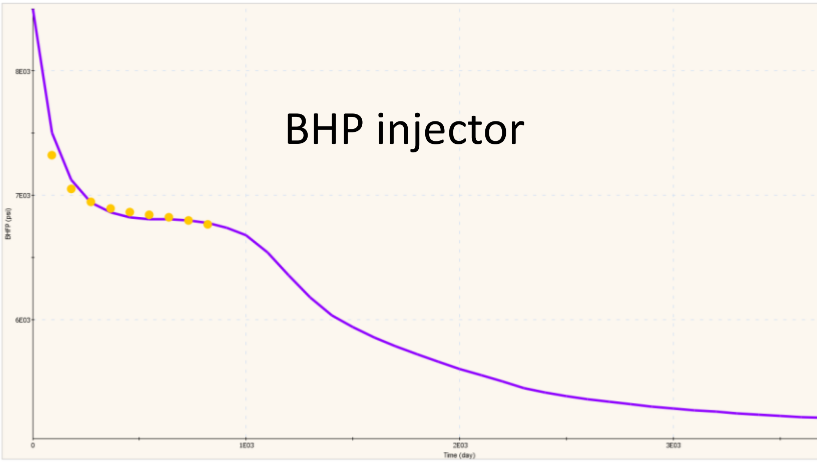
 

# HISTORY MATCHING

- **2 years ½ of production data**
  - Oil production rate
  - Gas production rate
  - BHP at injector well
- **10 years** : total simulation time

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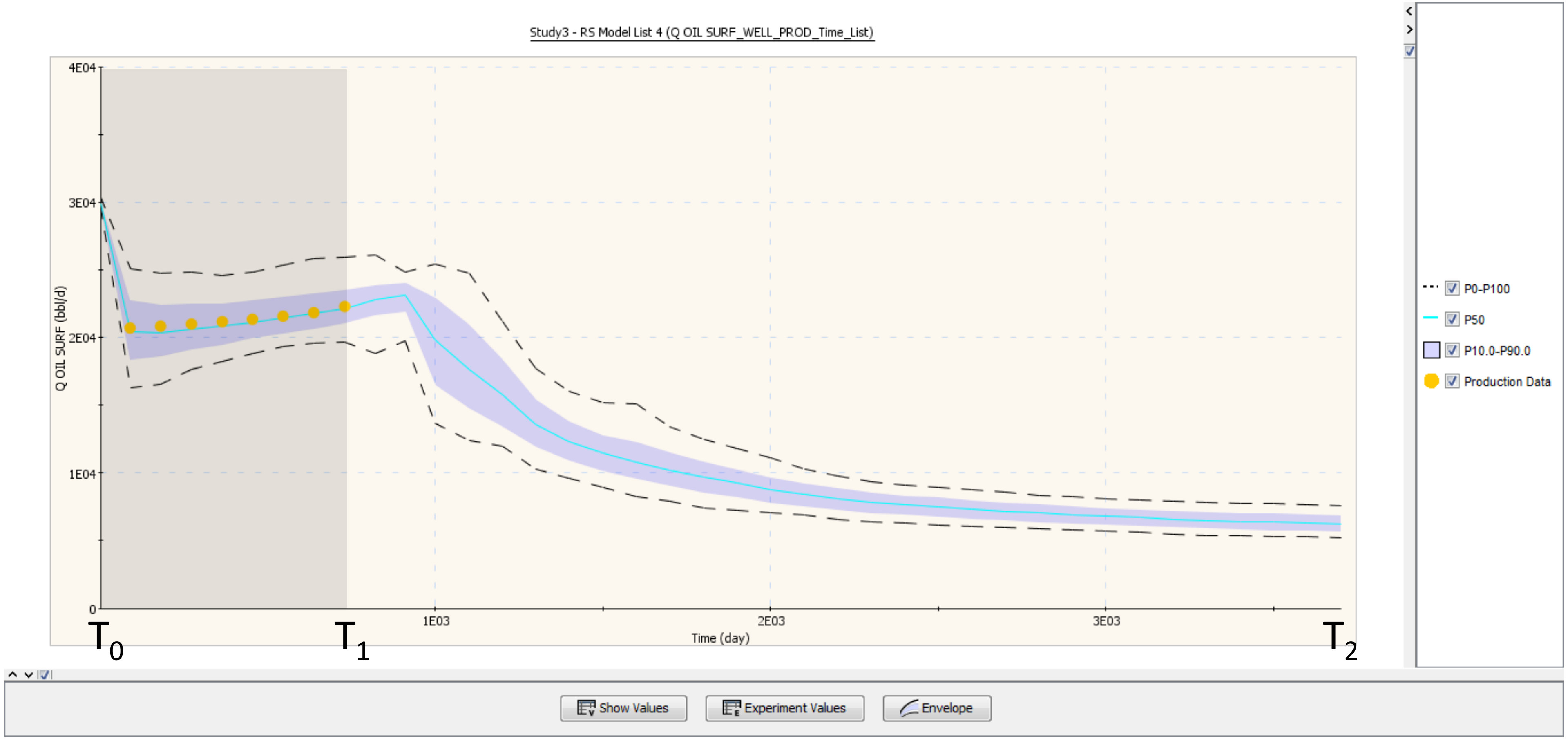
# BAYESIAN : MARGINAL POSTERIOR SAMPLING

$$p\left(X = x \middle| D_i^{obs} = d_i^{obs}\right)$$

# BAYESIAN : POSTERIOR PREDICTIONS

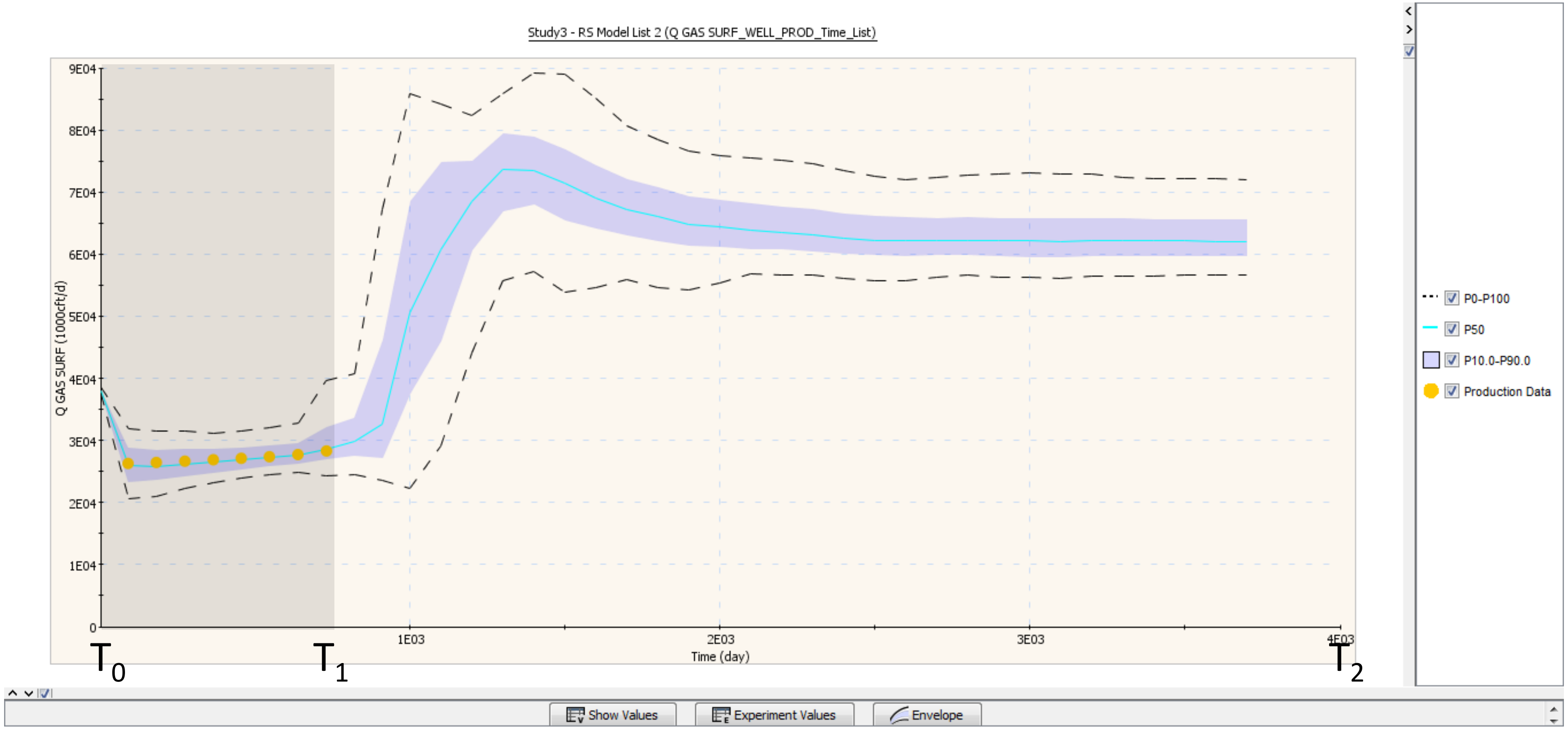
## Oil production rate



# BAYESIAN : POSTERIOR PREDICTIONS

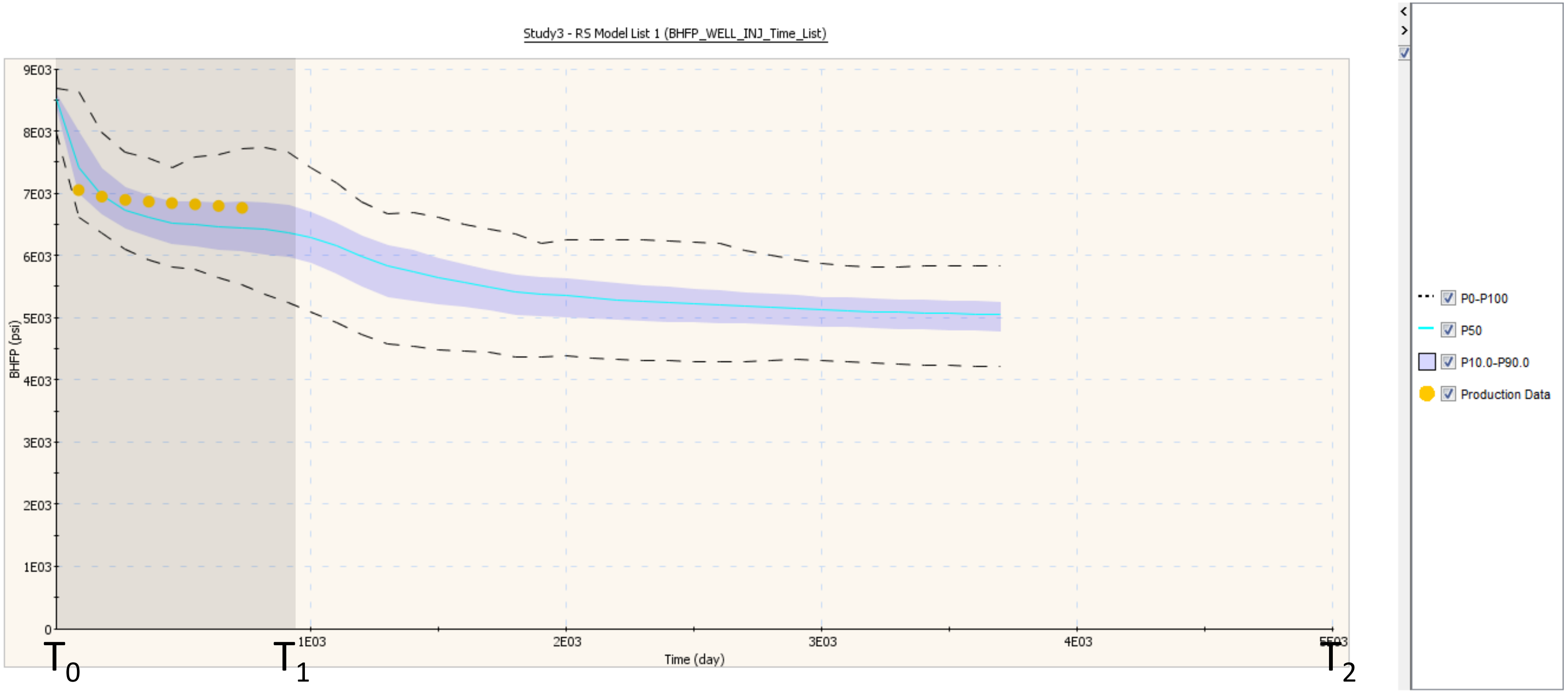
## Gas production rate



# BAYESIAN : POSTERIOR PREDICTIONS

## BHP Injector well



# FORECAST EXTREME SCENARIOS

- The solution of the History Matching (HM) optimization problem

$$\min_{x} \left\| G(x) - d^{obs} \right\|^2_{[T_0,T_1]}$$

  is only one solution among others that fit the production measures within fixed tolerances

- **Goal** : determine, among the HM solutions, the reservoir model which maximizes/minimizes the forecast production

➢ **Upper and lower bounds of the forecast production**

# FORECAST EXTREME SCENARIOS

- A **nonlinear constrained optimization** problem

$$\max_x \;/\; \min_x \; \mathrm{prod}(x)_{[T_0,T_2]}$$

$$s.t \; \left\| G(x) - d^{obs} \right\|^2_{[T_0,T_1]} \leq \epsilon_{abs}$$

Cumulated production on $[T_0, T_2]$

- Cumulated production could be replaced by (function of) other simulator responses, e.g. pressure at top of the reservoir

*Roggero and Guerillot, 1996*
*Delbos et al., 2010*

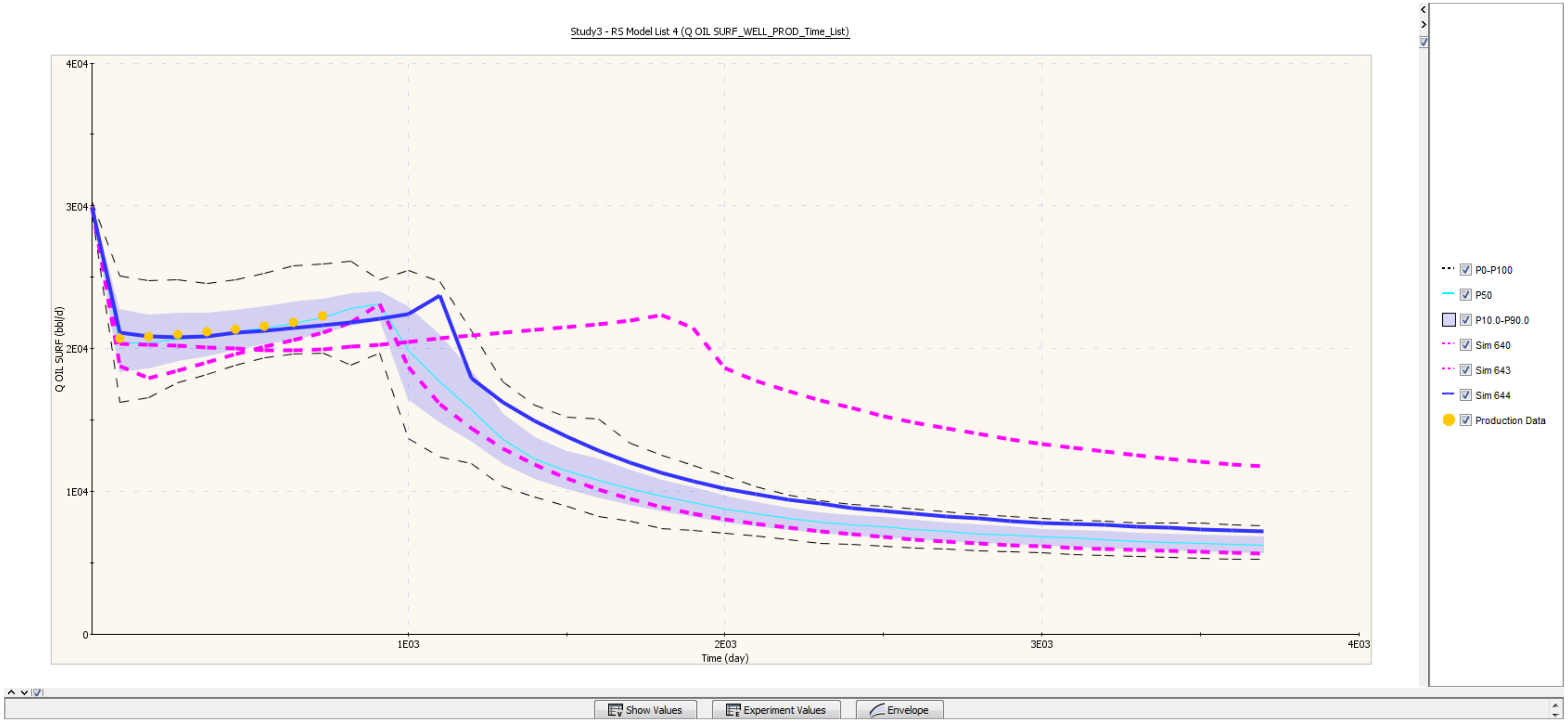
# FORECAST EXTREME SCENARIOS ON SURROGATES

- Extreme scenario
  - Surrogates of $\boldsymbol{F}$ and of $\mathrm{Prod}(.)_{[T_0,T_2]}$
  - **Nonlinear constrained optimization on surrogates**

$$\min_x / \max_x \widetilde{\mathrm{Prod}}_{T_2}(x)$$
$$\text{s.t. } \widetilde{F}(x) \leq \varepsilon$$

# FORECAST EXTREME SCENARIOS ON SURROGATES




- Accuracy of responses surfaces is not sufficient to compute extreme scenarios

# NONLINEAR CONSTRAINED DFO

DFO : Derivate Free Optimization

- Apply an **optimization method based on simulations**

- Dedicated algorithm for non linear constrained derivative free optimization
  **SQA = Sequential Quadratic approximation**

  *Langouët, Sinoquet*

  Interpolation-based trust region methods with local quadratic models

  *Powell, Conn, Scheinberg, Vicente, …*

# SQA METHOD : SEQUENTIAL QUADRATIC APPROXIMATION

Extension of NEWUOA (Powell) to constrained optimization

$$\min_{x} f(x)$$

**s.t.** $C_{DB}(x) \leq 0$ Derivative Based Constraints (linear / nonlinear)

$C_{DF}(x) \leq 0$ Derivative Free Constraints

- Constrained minimization sub-problems (SP)

$$\min_{\|d\| \leq \Delta} Q(x_k + d) \quad s.t. \begin{cases} C_{DB}(x_k + d) \leq 0, \\ \tilde{C}_{DF}(x_k + d) \leq 0. \end{cases}$$

- $Q$ and $\tilde{C}_{DF}$ are quadratic interpolation models of $f$ and $C_{DF}$ (black-box obj. function and constraints)

# SQA METHOD : SEQUENTIAL QUADRATIC APPROXIMATION

- **Initialization :**

  $x_0, f(x_0), C_{DF}(x_0), \Delta$

  $m$ interpolation points $(y_i, f(y_i), C_{DF}(y_i))_{i=1,m}$

- **At a given iteration k**
  - Build quadratic models $Q$ and $\tilde{C}_{DF}$
  - Minimization of the sub-problem (SP)
  - New simulation at $x_k + d^*$

    Define merit functions $\varphi$ and $\tilde{\varphi}$ : $\varphi(x) = f(x) + \sigma \sum_{i=1}^{n_{DF}} C_{DF}(x)^{\#}$
  - Validation of the new point with $R = \frac{\varphi(x_k) - \varphi(x_k + d^*)}{\tilde{\varphi}(x_k) - \tilde{\varphi}(x_k + d^*)}$
  - Model improvement step with a new simulation if $R < \eta$
  - Update the trust region radius $\Delta$

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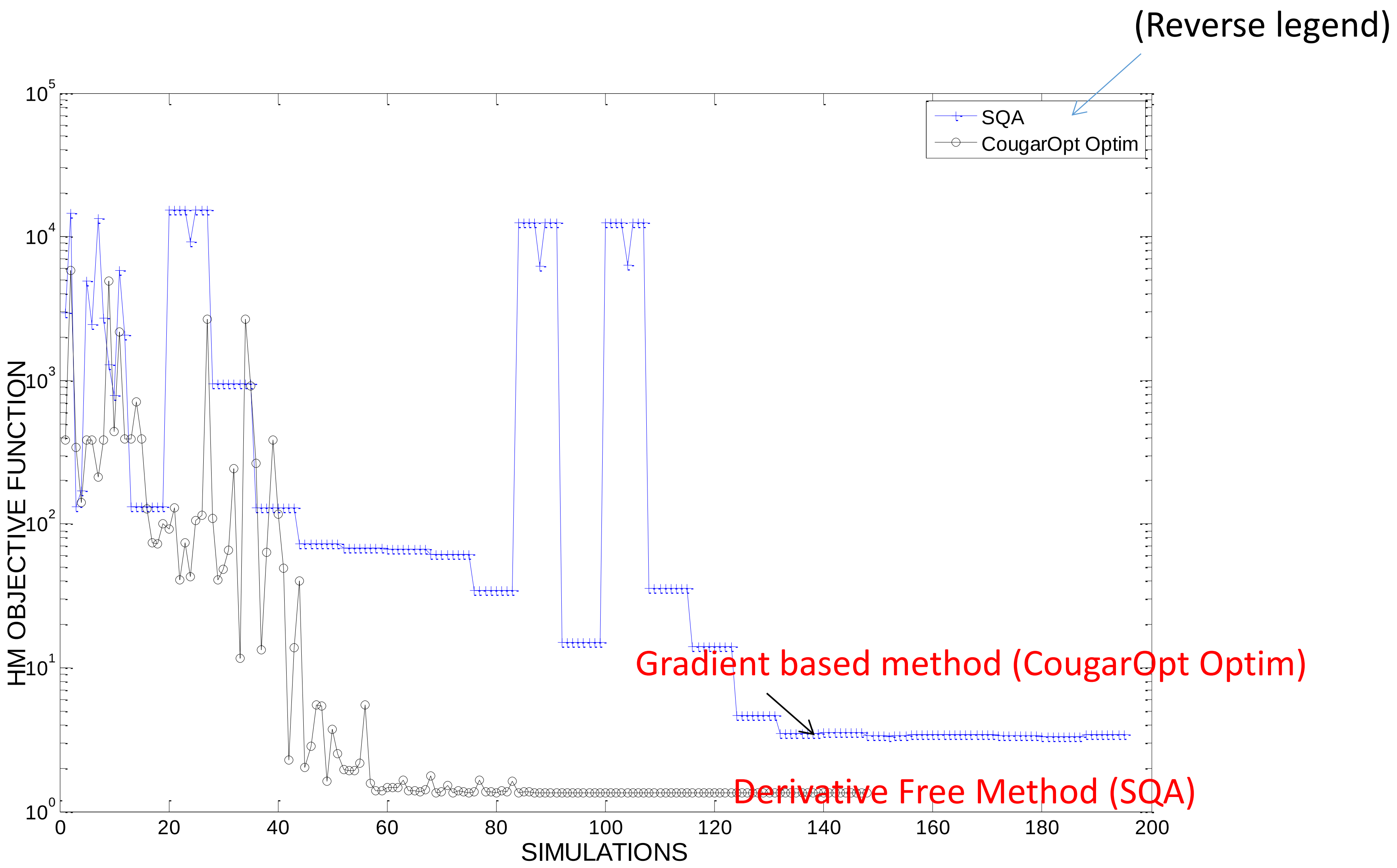
# HISTORY MATCHING WITH SQA

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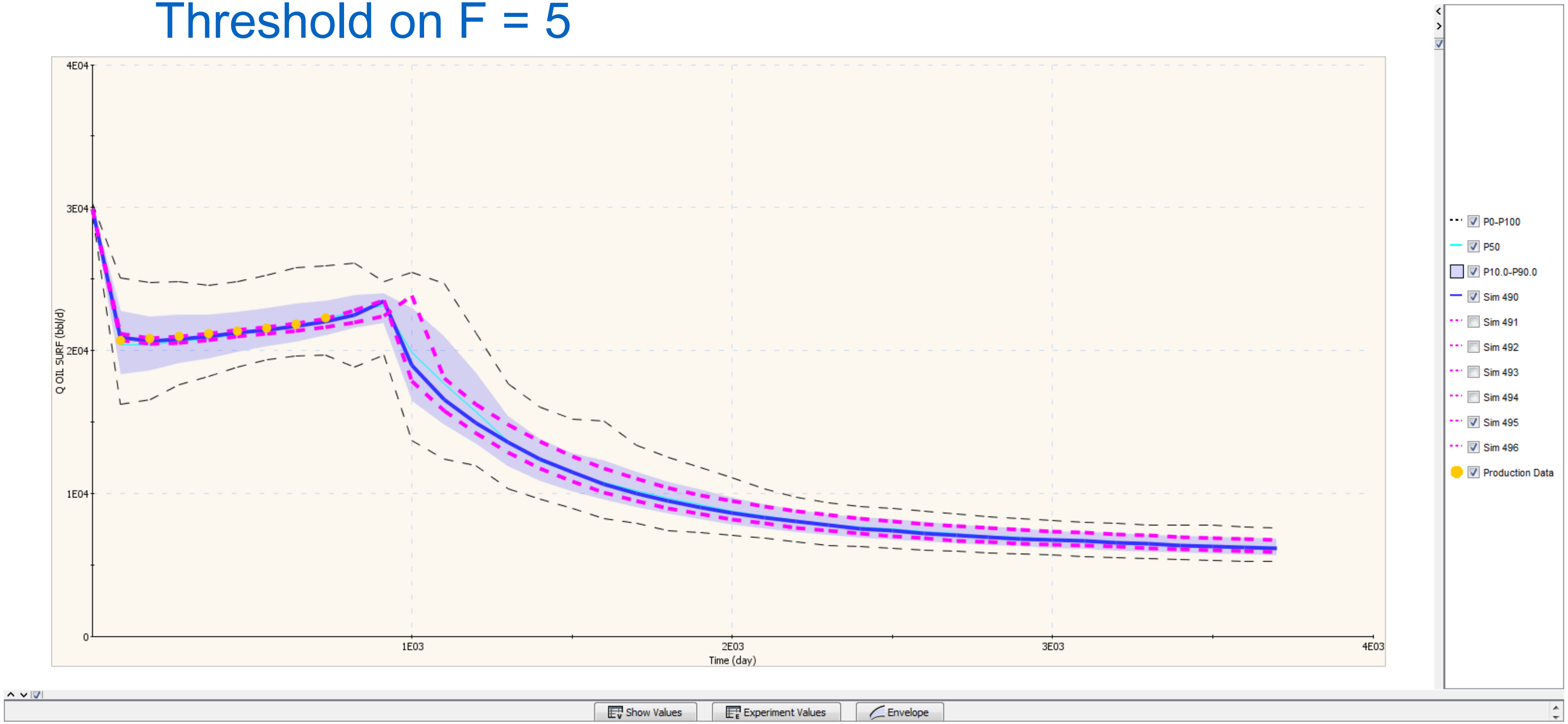
# EXTREME SCENARIOS OBTAINED WITH SQA

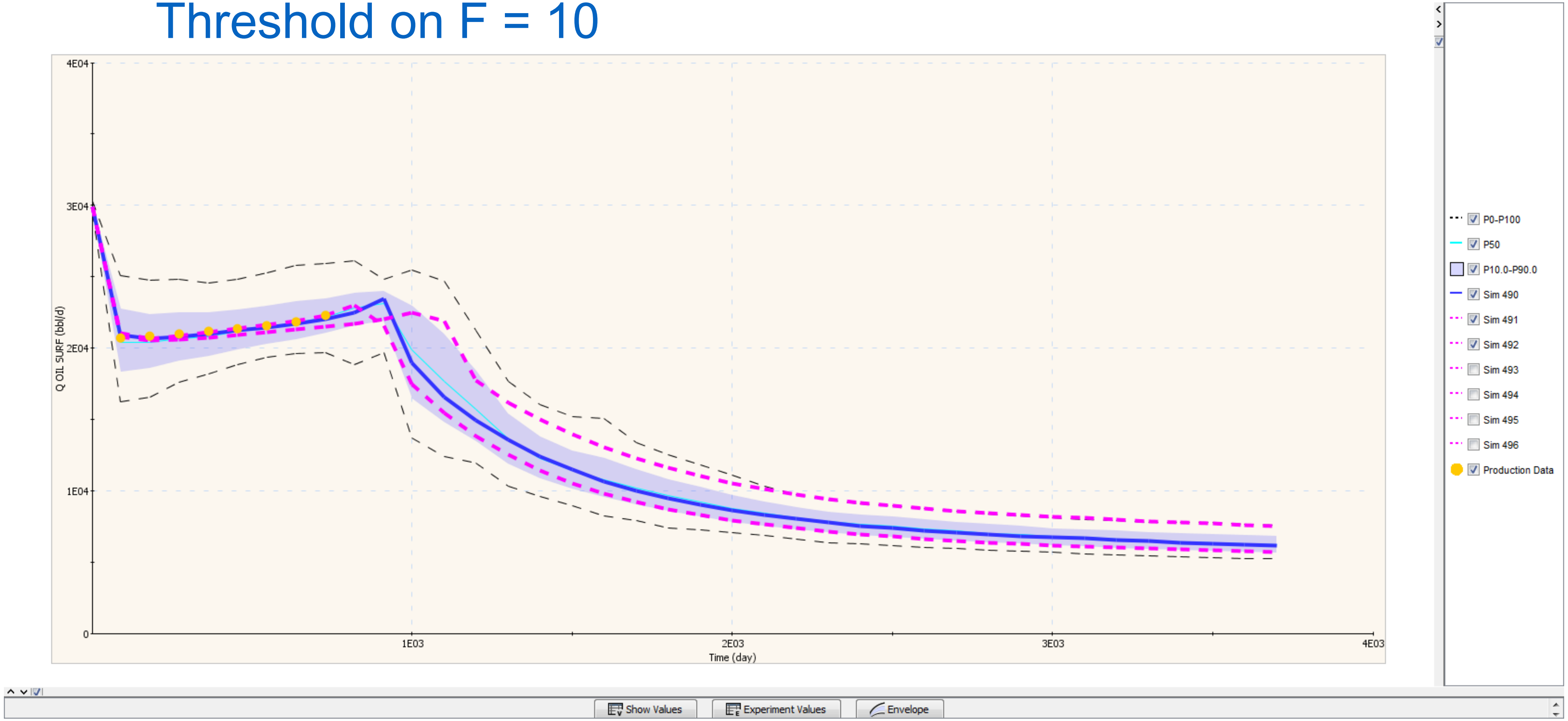
## Threshold on F = 5



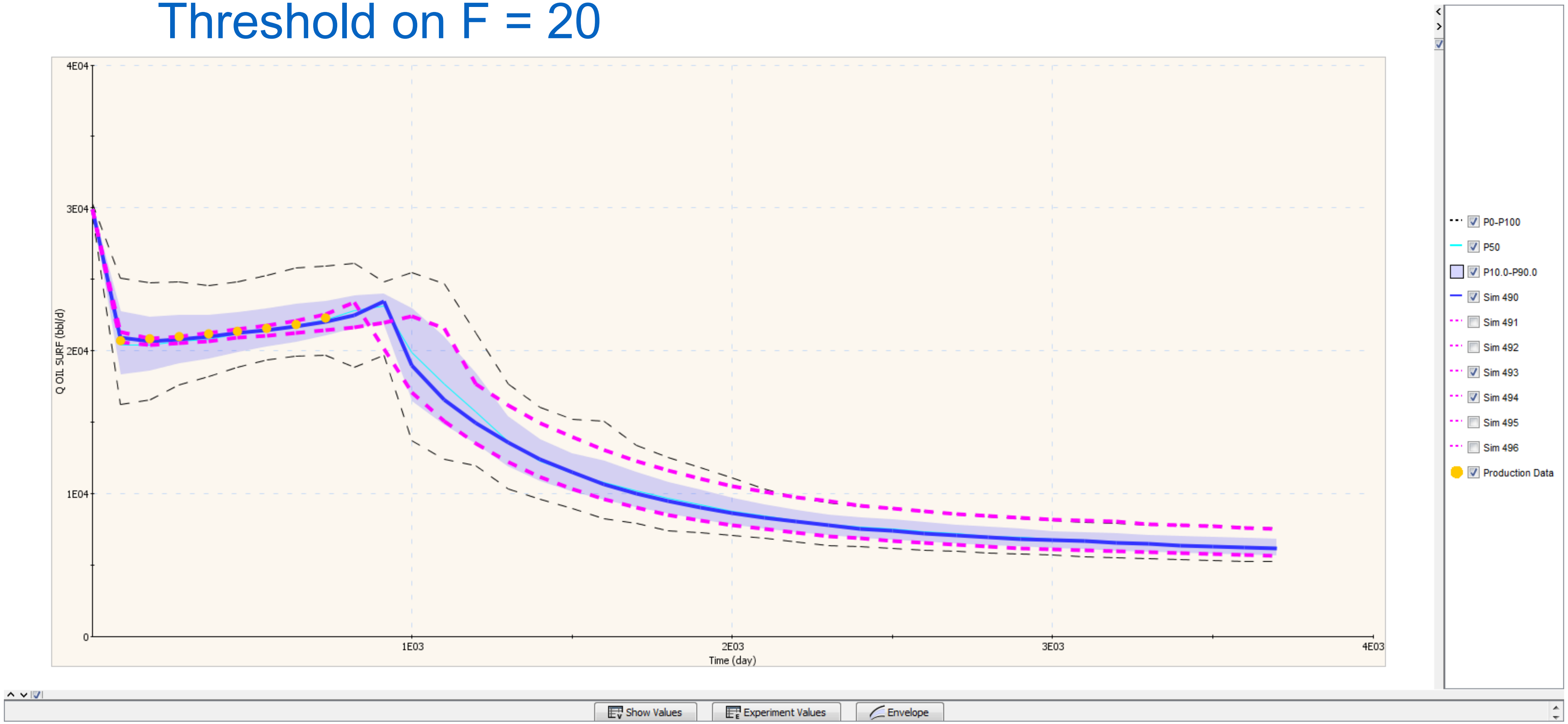
# EXTREME SCENARIOS OBTAINED WITH SQA

## Threshold on F = 10



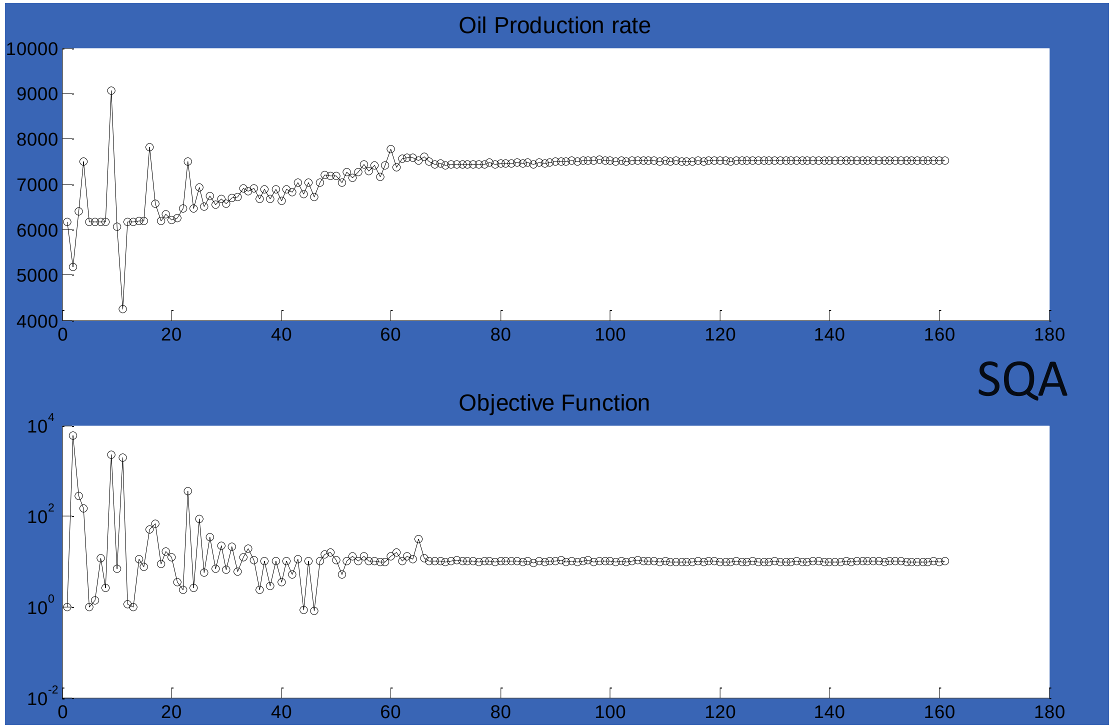
# EXTREME SCENARIOS OBTAINED WITH SQA

## Threshold on F = 20



# CONCLUSIONS

- Computing extreme scenarios with nonlinear constrained optimization based on simulations
  - with SQA (less than 80 simulations per optimization)
  - Could use adaptive surrogates if still too expensive

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# CONCLUSIONS

- **Computing extreme scenarios**
  with nonlinear constrained optimization based on simulations SQA:
  - $\sim$ 160 simulations needed
  - Some proposals:
    - Could use adaptive surrogates if still too expensive

- **Bayesian approach**
  - Full distribution/sampling of parameters and predictions is obtained
  - Need of accurate surrogate of $F$ in the whole parameters space or take into account the surrogate model error in the calibration
  - $\sim$ 500 simulations needed in propagation

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