3.10 BALANCED COUPLING BETWEEN VERTICAL MOTION AND DIABATIC HEATING FOR VARIATIONAL DATA ASSIMILATION

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ABSTRACT

A direct coupling between balanced vertical motion and diabatic heating within the Canadian North-American/Continental Meso4dvar analysis control vector is briefly described. It allows in principle the analyzed large scale diabatically forced divergent flow to be better specified at the initial time. Because of the basic assumptions on the characterization of “balance” we use, we only constrain a subset of vertical and horizontal modes of the full spectrum allowed in the analysis increment vector. The current procedure can potentially improve the coherent assimilation of precipitation data over organized mesoscale convective systems.

1. INTRODUCTION

Numerical weather prediction (NWP) models based on the primitive equations of motion admit solutions with a wide spectrum of frequencies ranging from fast inertia-gravity modes, slower internal gravity modes (important during deep convective events) and slower rotational modes. Proper adjustment of mass, winds and moisture during the analysis process is required in order to avoid projecting unrealistically large energy on strongly divergent modes at the initial time. At the Meteorological Service of Canada (MSC), a Limited-Area 4D-Var analysis system is currently being developed in order to enable the analysis of synoptic and mesoscale weather.

This system is referred to as Meso4dvar. The main objective of this new analysis system being the improved forecast of precipitation up to 48h, with a potential replacement of the Canadian Regional analysis system currently operational at the Canadian Meteorological Center (CMC). Figure 1 shows two types of limited area analysis/forecast domains. The large domain is for the “North American Continental” configuration. The Meso4dvar analysis is designed to suit the Limited-Area version of the GEM model (Cote et al. 1998). The latter being used operationally only in global mode with uniform and non-uniform mesh for medium-range and short-range weather forecasting respectively.

Tangent-linear (TL) and adjoint (AD) versions of GEM-LAM were developed as an extension of the work by Tanguay and Polavarapu (1999). Recently, the TL/AD GEM-LAM code was successfully coupled to the limited-area version of the variational analysis code.
Potted simply, the latter uses bi-Fourier spectral representation on a limited area domain rather than spherical harmonics on the sphere. Otherwise, the two configurations of the code were designed to be mostly transparent to the user. Helmholtz’s functions are being used in the two analysis systems with non-separable background error correlations in their respective spectral spaces. The horizontal resolution of the Nonlinear GEM-LAM NA-Continental model is set to 10 km. However, the horizontal resolution in the inner-loop of the incremental Meso4dvar is currently examined in the range 20-50 km. This represents a factor 5 increase in horizontal resolution as compared to the current inner loop of the global operational incremental 4D-Var system which is currently at T108.

The other two small analysis domains are for even higher resolutions around 2.5 km designed to improve short-range weather forecasting in the 0-12h range. The Meso4dvar code operated on this high-resolution grid was tested under simple observation assimilation experiments but there remains more work to introduce background-error correlations representative of errors at these small scales. Finally we note that due to the important connection between gravity waves and mesoscale convection, the use of diagnostic (as opposed to prognostic) balance relationships as described below may turn out to be inappropriate. Recent empirical results by Pagé and Zwack (2005) (see this volume, Sec. 1.24) at 2.5 km horizontal resolution with full physics clearly identify cases where a diagnostic relationship may still be advantageous.

2. DEFINITION OF ANALYSIS CONTROL VECTOR

In the variational analysis procedure, we analyze “analysis increments”; i.e. departures from a background field (a priori estimate):
\[ \delta \mathbf{x}_g = \mathbf{x} - \mathbf{x}_b \quad (2.1) \]

In order to make the “background term” or cost-function assume the simple quadratic form
\[ J_b = \frac{1}{2} \mathbf{\chi}^T \mathbf{\chi} \quad (2.2) \]

we make a change of analysis variables of the form:
\[ \mathbf{x} - \mathbf{x}_b = \mathbf{L} \mathbf{\chi} \quad (2.3) \]

s.t. \[ \mathbf{B} = \mathbf{L} \mathbf{L}^T \].
The transformation represented by the linear operator \( L \) is a composition of operators the reader can find in Derber and Bouttier (1999), Gauthier et al. (1999), Fisher (2003). Included in this transformation is a splitting of analysis-increments in terms of so called “balanced” and “unbalanced” components. Those are usually defined with simple linear approximation to more complex nonlinear balance equations; e.g. the local geostrophic assumption between mass and streamfunction.

3. BALANCED VERTICAL MOTION

In an attempt to generalize Fisher’s (2003) results, we only consider dynamical and physical balance relevant to the synoptic scales resolved by the N-A Continental Meso4dvar system. No attempt is made here to characterize similar diagnostic balance at the mesoscale.

Following the innovative work by Fisher (2003) at the European Center for Medium Range Weather Forecasts (ECMWF), we use the tangent-linear Quasi-Geostrophic (QG) Balance set of equations directly within the definition of the balanced part of the analysis vector. Fisher’s results showed improved synoptically coherent divergence increments w.r.t. their previous statistical regression approach. Higher order balance set of equations could also be envisaged as an extension. However, since this balance transform is currently applied at every simulations during the variational analysis minimization, and the fact that these involve elliptic equation solvers, higher-order balance may be too expensive to use operationally.

Additionally, even at first order QG Balance description, solving the required diagnostic equations is more expensive in a limited area context (as is the case here) as compared to global spectral models (such as ECMWF implementation). Currently, the procedure involves the linearization of the classical nonlinear balance equation (Charney 1955):

\[
\nabla^2 p_b = -\nabla \cdot (v_\psi \cdot \nabla v_\psi + f k \times v_\psi) \quad (3.1)
\]

and the QG-Omega equation:

\[
\frac{\nabla^2 \omega}{\frac{R_0^2}{\sigma p^*} \frac{\partial^2 \omega}{\partial \eta^2}} = R_{ad} \quad (3.2),
\]

where

\[
R_{ad} = \frac{f}{p^*} \frac{\partial}{\partial \eta} \left[ v_\psi \cdot \nabla (\nabla^2 \psi + f) \right] + \frac{R}{pp^*} \nabla^2 [v_\psi \cdot \nabla T] \quad (3.3)
\]

represents the adiabatic term contributing to the balanced vertical motion and \( \eta \) is the GEM-LAM/Meso4dvar vertical coordinate; \( p^* = p_s - p_t \) the difference in pressure between the surface and the model’s top. Define the auxiliary problem:

\[
\frac{d^2 \xi}{d \eta^2} = \frac{\sigma(\eta) p^*}{\frac{f}{2}} \xi; \quad (3.4)
\]

\[
\xi_{\eta=\eta_T} = \xi_{\eta=0} = 0.
\]

We use a finite-element representation of the fields in the vertical so that equation (3.4) defines a generalized eigenvalue problem \( R \eta \xi = \xi S \xi \). The solution in the vertical can thus be restricted to a subset of vertical modes using the latter defined vertical eigenvalue problem. Partial justifications for limiting the number of internal modes in the solution comes from the close relationship between QG Balance and the nonlinear normal mode initialization theory (more specifically Baer and Tribbia’s (1977) approach as shown on an f-plane by Leith 1980). The usual limitations of this set of equations in terms of dynamical balance description and practical application over tropical regions can be found in Daley (1991) for instance.

A global nonlinear normal mode approach rather than the QG set of equations may be more appropriate for global models in order to avoid these limitations. Discrepancies in the tropics between the two approaches in terms of low-frequency normal mode spectra and mode structure exist and can be found in Moura (1976).

We note that for the continental LAM domain considered above, the tropics appear essentially in the pilot and blending
zones used in the GEM-LAM and Meso4dvar analysis; i.e. the use of the QG balance equations remains in our case a viable option.

4. COUPLING BALANCED VERTICAL MOTION AND DIABATIC HEATING

We consider a diabatic extension of ECMWF’s approach. We couple balanced vertical motion at the synoptic scales and the parameterized diabatic heating within a 3D/4D-Var approach. The extension to the mesoscale problem (assuming some constraining equation on vertical motion and explicit diabatic heating similar to the synoptic scales can be found) could possibly follow a similar approach in principle. The presence of diabatic heating influences the balanced vertical motion according to

\[ \mathbf{L} \, \omega = \nabla^2 \omega + \frac{f_0}{\sigma p^*} \frac{\partial^2 \omega}{\partial \eta^2} = (4.1) \]

\[ = R_{ad} + R_{\text{diab}} \]

The QG omega operator \( \mathbf{L} \) is assumed linear in terms of control variables. This is not exactly true since the static stability \( \sigma \) depends on the vertical temperature gradient and also on the humidity field. We recognize that this stability effect on the balanced omega field may be important but we neglect this effect for the moment and will come back to it in a future study. The diabatic term is

\[ R_{\text{diab}} = -\frac{R}{p c_p} \nabla^2 \frac{dT}{dt} \text{phys} \quad (4.2) \]

For illustration purposes and in a similar way as Fillion and Errico (1997) (hereafter referred to as FE97), over organized deep convection, we simply estimate the time tendency from physical processes as the effect from parameterized deep convection as used in the numerical model; i.e.

\[ \left. \frac{dT}{dt} \right|_{\text{phys}} \equiv C(T, q) \quad (4.3) \]

where \( C \) is the nonlinear convective parameterization scheme. The latter may depend on the control variables via temperature \( T \) and specific-humidity \( q \) vertical profiles. Arguing as in Fisher (2003), the need to keep a linear relationship in the definition of the control variable means we linearize the diabatic omega equation around the basic state (bold quantities represent linear or linearized operators in the following):

\[ \mathbf{L} \delta \omega_b = R_{ad} - \frac{R}{p c_p} \nabla^2 K(\delta T, \delta q) \quad (4.4) \]

\[ \delta \omega_b = \mathbf{L}^{-1} \left[ R_{ad} (\delta \psi, \delta T) - \frac{R}{p c_p} \nabla^2 K(\delta T, \delta q) \right] \]

\[ \delta \chi_b = \mathbf{J} \delta \omega_b \text{ Continuity/Helmholtz's Equation (4.5)} \]

N.B.: Triggering issues based on the vertical velocity field (such as in Kain-Fritsch’s moist convective parameterization scheme used at MSC) are not considered in the formulation here. Thus, in theory, the combined use of (4.4) and (4.5) gives us a diagnostic relationship for \( \delta \chi_b \); i.e.

\[ \delta \chi_b = \mathbf{D} \delta \omega_b \quad \text{Continuity/Helmholtz's Equation (4.6)} \]

We note in passing that the above equation refers to the balanced part of the analysis increment and emphasizes the notion of “balanced moisture” increments. In practice, equation (4.6) represents a difficulty (to say the least) when computing a horizontal Laplacian of the convective heating field. Deep convective parameterization schemes as used operationally in most NWP models (for horizontal resolutions above say 10 km), operates in vertical columns and are applied only at model grid points where some triggering conditions need to be satisfied.

Apart the fact that these schemes are strongly nonlinear and discontinuous (i.e. hardly linearizable, ref. Fillion and Bélair 2004), taking a Laplacian of these convective temperature tendencies results in a noisy field that requires spatial filtering. This filtering was used successfully by Pâgé and Zwack (2005) (this proceeding, Sec 1.24). Also, the incorporation of additional physical processes in the evaluation of the physical time tendencies (with some spatial
filtering) can represent a less severe problem than the simple use of convective tendencies as considered here for illustration purposes only.

As with the use of the adiabatic QG balance and Omega equation in ECMWF’s analysis, we expect the careful use of the diabatic extension of these balance equations (using the numerical model physical parameterization) to improve the effective background error covariance model over dynamically and diabatically active regions.

This improved covariance model was identified as one important ingredient impacting the vertical structure of analysis increments over deep convective regions when precipitation assimilation is considered for instance (ref. FE97). The proposed formulation above also represents a partial solution to slow timescale imbalances examined in Fillion (2002).

5. ACKNOWLEDGEMENTS

Thanks are due to Michel Desgagné & Jean Coté and Vivian Lee at RPN for their help with their scientific formulation of GEM-LAM and its practical use respectively. Thanks also to Yves Chartier from RPN for his help with numerous details related to the particular GEM-LAM geometry and file structure.

6. REFERENCES


