2.27 DEVELOPMENT OF A STOCHASTIC PRECIPITATION NOWCAST SCHEME FOR FLOOD FORECASTING AND WARNING

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1. BACKGROUND

Met Office research has shown that there is little to be gained from the further refinement of deterministic extrapolation algorithms for precipitation nowcasting (Bowler and Pierce, 2002). Thus, recent R&D has focused on the development of a stochastic scheme capable of quantifying the uncertainties and errors (hereafter, referred to simply as error) in precipitation nowcasts and conveying these to the customer. In pursuing this approach, it is recognised that whilst there will undoubtedly be further improvements in the deterministic Numerical Weather Prediction (NWP) of precipitation through, for example, the introduction of higher resolution NWP models, improved model formulation and data assimilation, these will best be exploited within a framework that seeks to quantify the associated errors that will always be an intrinsic part of precipitation observation and forecasting.

2. OVERVIEW OF STEPS

In keeping with the nowcasting methodology employed in Nimrod (Golding, 1998) and Gandolf (Pierce et al., 2000), the Short-Term Ensemble Prediction System (STEPS) blends an extrapolation forecast with a deterministic NWP forecast of precipitation. In Nimrod and Gandolf this blending is achieved in physical space and the weight given to the extrapolation component takes the form of a fixed exponential decay with time. In STEPS, the merging of the extrapolation and NWP component forecasts is performed in a scale-dependent way using a cascade model framework (see 2.1 below). This allows the scheme to capture the scale dependent loss of predictive skill in the extrapolation forecast with advancing lead time (Venugopal et al., 1999; Germann and Zawadzki, 2002) and the scale-dependence of the skill of the NWP forecasts. The weights assigned to the extrapolation and NWP components are computed on-line for each STEPS run. This ensures that optimum use is made of both extrapolation and NWP models as the meteorology of a precipitation event evolves and the skill of the models changes.

An ensemble of nowcasts is generated by blending the extrapolation and NWP component forecasts with noise whose spatio-temporal statistical properties are derived from those of recent weather radar based analyses of instantaneous rain rate (hereafter referred to as rain analyses). Uncertainties in the advection and Lagrangian evolution of the extrapolation nowcast are modelled. Also, some account is taken of errors in the NWP component, although, at present this is limited to: a bias correction applied to the NWP model precipitation fields; the merging of the advection and NWP diagnosed precipitation velocity fields makes some adjustment for displacement/timing errors in the NWP forecast; the weight assigned to the NWP component will vary according its skill as measured against the latest rain analysis.

A single forecast realisation is a blend of three cascades: extrapolation, NWP and noise. A set of weights are calculated for each of these forecast components, for each cascade level. The weights applied to the extrapolation component are derived from a hierarchy of second order Auto-Regressive (AR-2) models; those ascribed to the NWP forecast are based upon its skill as measured by cross correlating the latest rain analysis with a time synchronous NWP forecast. The weights are formulated such that, for each level, they preserve the variance of the level rather than sum to one since the mean of each level has been set to zero.

This model formulation ensures that, in each ensemble member, extrapolated features are progressively replaced by noise from the smallest scales upwards. This process proceeds at a rate that is consistent with the diagnosed temporal persistent of features on each level in the cascade, but is arrested by the predictive skill of the NWP model. Figure 1 shows an example of a single member from a STEPS ensemble forecast.

* A distinction is made between uncertainties such as those inherent in the remote sensing of precipitation using weather radar, and errors such as those arising from the use of imperfect models to diagnose and predict surface rain rate.

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2.1 Cascade representation of precipitation fields

STEPS exploits a cascade representation of precipitation fields. This provides a suitable framework for modelling their fractal and dynamic scaling properties (see Lovejoy et al., 1996; Venugopal et al., 1999).

A field of instantaneous rain rate (estimated from radar or generated by a NWP model), with dimensions of \( L \) by \( L \) pixels, can be decomposed into a hierarchy of component fields representing variability on a discrete set of horizontal scales. Expressed in units of rain rate (e.g., mm h\(^{-1}\)), the cascade is multiplicative and takes the form:

\[
R_{i,j}(t) = \prod_{k=1}^{n} X_{i,k,j}(t) \quad \text{for} \quad i=1,\ldots,L, \quad j=1,\ldots,L, \quad L=2^n
\]  

where \( n \) is the number of component levels in the cascade, and the \( k^{th} \) field in the cascade, \( X_k(t) \), represents the variability in the original field with frequencies, \( \omega_k \), in the range \( \frac{2^{k-1}}{L}, \frac{2^k}{L} \) pixel\(^{-1} \) at time, \( t \). However, it is more convenient from a modelling perspective to work in units of 10+10log\(_{10} R \) because the log transformation reduces the skewness of the probability distribution making it closer to a Normal distribution, and the addition of 10 reduces the truncation effect for very light rain rates. Expressed in units of decibels of rain rate, dBR (10 log\(_{10} R \)), the cascade becomes additive

\[
dBR_{i,j}(t) = \sum_{k=1}^{n} X_{i,k,j}(t) \quad \text{for} \quad i=1,\ldots,L, \quad j=1,\ldots,L, \quad L=2^n
\]

where \( n=8 \) in STEPS.

A field of surface rain rate expressed in dBR is transformed into the frequency domain by means of a Fast Fourier Transform (FFT). Each level, \( X_k(t) \), of the cascade is calculated using a band-pass filter based upon a Gaussian window to pass the appropriate frequencies. An inverse transform is performed to return each Fourier component of the original field back into the spatial domain. Finally, the cascade levels are normalised using

\[
Y_{i,k,j}(t) = \frac{X_{i,k,j}(t) - \mu_k(t)}{\sigma_k(t)}
\]

where \( \mu_k(t) \) and \( \sigma_k(t) \) are the mean and standard deviation of the \( k^{th} \) level respectively. The \( Y_{i,k,j}(t) \) are referred to as the component levels of the cascade.

2.2 Lagrangian temporal evolution and associated errors

STEPS models the evolution of the precipitation field by blending the extrapolation forecast with an NWP forecast and noise. The noise has statistical properties inferred from rain analyses and serves several purposes: its addition accounts for errors in the Lagrangian evolution of the extrapolation nowcast; it adds power to the NWP forecast at scales where precipitation structures are inadequately resolved.

The method employed to model errors in the field evolution reflects the need to account for the observed dependency of the rate of evolution on the horizontal length scale of precipitation features. Field evolution is modelled separately on each level in the cascade in a Lagrangian reference frame. A hierarchy of second order Auto-Regressive (AR-2) models – one for each cascade level – is used to establish the rate of evolution as function of scale (Seed, 2003)

\[
Y'_{i,k,j}(t+t_l) = \phi_{1,i,j}(t) \cdot Y'_{i,k,j}(t+t_l-\Delta t) + \phi_{2,i,j}(t) \cdot Y^n_{i,k,j}(t+t_l-2\Delta t)
\]

where \( Y'_{i,k,j}(t+t_l) \) is the predicted extrapolation cascade valid at lead time, \( t_l \), based upon similar cascades valid at the two previous time steps. The AR-2 parameters, \( \phi_{1,i,j}(t) \) and \( \phi_{2,i,j}(t) \), control the rate of evolution at each scale, and are determined from a temporal sequence of three rain analyses spanning a 30 minute time period.

The purpose of this approach is to limit the useful range of the extrapolation forecast in a scale dependent fashion in accordance with an objective measure of forecast skill. Beyond this useful range, the statistical properties of the extrapolated field are preserved by replacing extrapolated features at the relevant scales with a randomly generated field exhibiting the radar inferred spatial and temporal correlation structures.

To achieve this, a separate cascade of stochastic noise is maintained

\[
Y'^{n}_{i,k,j}(t+t_l) = \phi_{1,i,j}(t) \cdot Y'^{n}_{i,k,j}(t+t_l-\Delta t) + \phi_{2,i,j}(t) \cdot Y^n_{i,k,j}(t+t_l-2\Delta t) + \phi_{3,i,j}(t) \cdot \epsilon_{i,k,j}(t+t_l)
\]

where \( Y'^{n}_{i,k,j}(t+t_l) \) is the predicted noise cascade valid at lead time, \( t_l \), \( \epsilon_{i,k,j}(t+t_l) \) is a cascade of temporally independent but spatially correlated noise generated for each time step. Thus, a cascade of temporally correlated noise is maintained for the duration of the forecast. Both this cascade and the extrapolation cascade are advected using the current estimate of the velocity field (see 2.3 below).

The noise fields are chosen to be Gaussian (Menabde et al., 1999), \( N[0,1] \), and \( \phi_{3,i,j}(t) \) is given by (Salas et al., 1980)
\[ \phi_{k,t}(t) = \frac{1 + \phi_{k,t}(t)}{1 - \phi_{k,t}(t)} \left[ \phi_{k,t}(t) \right] \] (6).

This ensures that \( Y_{\text{prop},k,t}(t + t^*) \) has the same normalisation as \( Y_{\text{prop},k,t}(t) \) (mean zero and variance 1).

The field generated by the cascade is non-zero everywhere by construction, so a threshold is applied to set the very low rain rates to zero.

### 2.3 The advection scheme and advection error

A backwards-in-time advection scheme based upon a solution of the optical flow constraint equation is employed (Bowler and Pierce, 2002).

The uncertainty in the motion of the precipitation is modelled by adding a perturbation to the diagnosed advection velocity field. This is statistically homogeneous in space and uncorrelated with the forecast velocity (Bowler et al., 2003)

\[ v_{\text{perturbation}}(t + t^*) = f(t) v_{\text{perturbation}}(t) \] (7).

Validation studies have shown that the advection error has a symmetric exponential distribution. Thus, \( f(t) \) in (8) takes the following, empirically derived forms for components parallel and perpendicular to the forecast velocity

\[ f_{\text{parallel}}(t) = -7.68 + 10.88 \cdot t^*^{0.23} \] (8)

\[ f_{\text{perpendicular}}(t) = -2.72 + 5.76 \cdot t^*^{0.31} \] (9).

The perturbed velocity field, \( v_{\text{noisy}}(t + t^*) \), for lead time, \( t + t^* \), is given by:

\[ v_{\text{noisy}}(t + t^*) = v_{\text{smooth}}(t) \left[ 1 - \frac{5}{60} f \right] + v_{\text{perturbation}}(t + t^*) \] (10)

where \( f(x) \) is a bias removal required to account for the fact that the diagnosed extrapolation velocity field is not an unbiased predictor of future velocity (ibid., 2003). \( v_{\text{smooth}}(t) \) is the diagnosed extrapolation velocity field after temporal smoothing to remove measurement noise (ibid., 2003).

At each forecast time step the extrapolation and noise cascades must be advected because their evolution is modelled in a Lagrangian reference frame. The velocity field used must reflect the diagnosed extrapolation velocity field at short lead times and the apparent velocity of the NWP precipitation fields at longer range. Thus, a merged velocity field is computed:

\[ v(t + t^*) = w_{\text{prop}}^f(t + t^*) v_{\text{prop}}(t + t^*) + w_{\text{prop}}^n(t + t^*) v_{\text{noisy}}(t + t^*) \] (11)

where \( w_{\text{prop}}^f(t + t^*) \) and \( w_{\text{prop}}^n(t + t^*) \) are the weights assigned to the extrapolation and NWP velocity fields respectively, and the subscript, 2, refers to the second level of the cascade. These weights are identical to those used in the merging of the extrapolation and NWP forecast cascades as described in 2.4 below. The weights for the second level of the cascade are used because the climatological values of the auto-correlation coefficients at this level most closely match the measured auto-correlation of the velocity field.

### 2.4 Combining extrapolation, NWP and noise cascades

A single forecast realisation is produced by blending the extrapolation and stochastic noise cascades described above with an NWP model forecast cascade

\[ Y_{\text{prop},k,t}(t + t^*) = w_{\text{prop}}^f(t + t^*) Y_{\text{prop},k,t}^f(t + t^*) + w_{\text{prop}}^n(t + t^*) Y_{\text{prop},k,t}^n(t + t^*) \] (12)

where \( w_{\text{prop}}^f \), \( w_{\text{prop}}^n \), \( w_{\text{prop}}^m \) are the weights given to the extrapolation, noise and NWP model cascades respectively. These weights are calculated from the estimated skill of the extrapolation and NWP model forecasts as follows. The fraction of the variance explained by each forecast component is used as a measure of it’s forecast skill. For the NWP component this is defined as

\[ \chi^2_k = 1 - \frac{\text{MSE}_k}{\sigma_k^2} \] (13)

where \( \text{MSE}_k \) is the mean squared error for the \( k^{\text{th}} \) level in the NWP cascade when compared with a time synchronous rain analysis, and \( \sigma_k^2 \) is the variance of the \( k^{\text{th}} \) level in the rain analysis cascade. In the case of the extrapolation forecast, the forecast error is calculated iteratively for each time step in the forecast sequence using the AR(2) parameters, assuming that the error in the radar rainfall estimates is independent of scale and is fixed at 15% of the field variance

\[ \chi^2_k(t + t^*) = \frac{\chi^2_k(t + t^*)^2}{1 - \chi^2_k(t + t^*)} \] (14).

This is clearly very optimistic for the smaller scales and further work is planned to improve the representation of the radar measurement errors in STEPS. The minimum skill for both extrapolation and NWP forecasts is set at 0.1 so as to keep the weights sensible in situations with very low skill for either forecast.
The variance of $z$, a weighted sum of two random numbers $x$ and $y$ is given by

$$\sigma_z^2 = a^2 \sigma_x^2 + b^2 \sigma_y^2 + 2ab \sigma_{xy}$$  \hspace{1cm} (15).

This can be evaluated if $x$ and $y$ are normalised to have zero mean and unit variance. When blending the extrapolation and NWP component forecasts in STEPS the skill of each forecast component can be used to estimate the covariance term

$$\sigma_{xy} = \alpha' \alpha$$  \hspace{1cm} (16).

Hence, the variance of the combined extrapolation and NWP forecast must reflect the skill of the combination.

The noise term is added in a way that represents the uncertainty in the blended forecast and preserves the variance of the combination of the three terms in (12).

The weights for the three cascades are calculated as follows for scales and lead times where the extrapolation has a higher skill than the NWP forecast

$$w_x^e(t + t_f) = \frac{\lambda_x^e(t + t_f)^2}{\lambda_x^e(t + t_f)^2 + \lambda_y^e(t + t_f)^2(1 - \lambda_x^e(t + t_f))^2 + 2\lambda_x^e(t + t_f)(1 - \lambda_x^e(t + t_f))\lambda_y^e(t + t_f)}$$  \hspace{1cm} (17)

$$w_y^e(t + t_f) = \frac{\lambda_y^e(t + t_f)^2}{\lambda_x^e(t + t_f)^2 + \lambda_y^e(t + t_f)^2(1 - \lambda_x^e(t + t_f))^2 + 2\lambda_x^e(t + t_f)(1 - \lambda_x^e(t + t_f))\lambda_y^e(t + t_f)}$$  \hspace{1cm} (18)

For situations where the NWP skill is greater than the extrapolation, the weights are calculated using

$$w_x^n(t + t_f) = \frac{\lambda_x^n(t + t_f)^2}{\lambda_x^n(t + t_f)^2 + \lambda_y^n(t + t_f)^2(1 - \lambda_x^n(t + t_f))^2 + 2\lambda_x^n(t + t_f)(1 - \lambda_x^n(t + t_f))\lambda_y^n(t + t_f)}$$  \hspace{1cm} (19)

$$w_y^n(t + t_f) = \frac{\lambda_y^n(t + t_f)^2}{\lambda_x^n(t + t_f)^2 + \lambda_y^n(t + t_f)^2(1 - \lambda_x^n(t + t_f))^2 + 2\lambda_x^n(t + t_f)(1 - \lambda_x^n(t + t_f))\lambda_y^n(t + t_f)}$$  \hspace{1cm} (20)

The weight given to the noise term is then given by

$$w_n(t + t_f) = \frac{w_x^n(t + t_f)^2 + w_y^n(t + t_f)^2 + 2w_x^n(t + t_f)w_y^n(t + t_f)\lambda_y^n(t + t_f)}{2w_x^n(t + t_f)w_y^n(t + t_f)\lambda_y^n(t + t_f)}$$  \hspace{1cm} (21).

During the forecast evolution, forecast skill is regressed towards a climatological skill (Bowler et al., 2003). In a similar way, the values of the AR-2 model parameters are also regressed towards climatological values. These climatologies are 30 day rolling values computed on-line.

Since the levels in the cascade are independent, the variance of the output field is simply the sum of the variances of the cascade levels. The cascade level variance, $\sigma_v^2$, is assumed to follow a power law. The scaling exponent can be estimated from the most recent rain analysis, and the variance for each level in the cascade calculated for each time step in the forecast.

The output forecast field (in dBR units) is generated using

$$dBR_{(t + t_f)} = \mu(t + t_f) + \sum_{i=0}^{n} \sigma_i(t + t_f) Y_{i,j}(t + t_f)$$  \hspace{1cm} (22).

The field in units of dBR is then converted back to rainfall rate and a threshold of 0.1 mm h$^{-1}$ is applied.

### 2.5 Performance of STEPS

A preliminary evaluation of STEPS indicates that its extrapolation forecast is likely to perform, on a par with the Met Office’s existing, operational precipitation nowcasting schemes, Nimrod and Gandolf (they use similar advection methods). Statistics compiled using one month’s worth of data (see figure 2) show that the scheme produces a skilful forecast of the probability of precipitation. An extended, parallel operational trial of the scheme will commence at the Met Office and Australian Bureau of Meteorology in autumn 2005. Further research and development is underway to model the uncertainty in the radar estimates of rain rate, and devise a more rigorous treatment of NWP model errors.

### 2.6 Applications of STEPS: Improving predictions of fluvial flows and floods

Current operational flood forecasting practice involves ingesting a best estimate of catchment average rainfall from available observations and forecasts into a lumped rainfall-runoff model to produce a single forecast of river flow. No uncertainty estimates are provided, and flood warnings are issued on the basis of whether or not the hydrologist considers it likely that a threshold flow or river level will be exceeded. This judgement is based upon a range of information including a perception of model reliability derived from past experience.

The availability of an ensemble of rainfall forecasts offers the opportunity to generate multiple forecasts of river flow from a given time origin, and thereby quantify both the accuracy of the flow forecast and the likelihood of a given flow threshold being exceeded (Moore, 2002). Since the uncertainty in the rainfall forecast is likely to dominate flow forecast accuracy, ignoring other sources of error such as those associated with the rainfall-runoff
model itself, should provide at least a first order indication of forecast uncertainty.

Above a threshold flow, costs will begin to be incurred as damage to property and social impacts become a possibility. Decision theory (for example, see: Raiffa and Schlaifer, 1961; Krzysztofowicz, 2001) allows a loss function to be combined with the forecast flow PDF to obtain the PDF of damage costs. Whilst theoretically straightforward, the calculation of loss functions quantifying damage costs at different flow levels may not be easy. However, reasonable approximations can be made, thus allowing the problem of when to issue a warning to be placed on a firmer scientific footing.

3. REFERENCES


Figure 1 Comparison of radar-based rain analyses valid at the data times of 0100 GMT (top-left) and 0130 GMT (top-right) on 20/05/05, with a single ensemble member of a T+30 minute STEPS forecast of rain rate (bottom-left), and a time synchronous T+90 minute NWP forecast of rain rate (bottom-right) generated by an experimental ~4km NWP model, valid at 0130 GMT on 20/05/05.
Figure 2 Brier skill scores for STEPS based upon one month’s worth of data, here presented as a function of forecast range (X axis), and precipitation rate (four lines). A score greater than zero demonstrates skill above that expected of climatology. The four lines show STEPS to be more skilful than climatology for lead times in excess of six hours.